A DISTANCE-INDEPENDENT INDIVIDUAL TREE BASAL AREA GROWTH MODEL FOR NATURAL EVEN-AGED STANDS OF SHORTLEAF PINE IN EASTERN OKLAHOMA AND WESTERN ARKANSAS

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PREFACE

Several distance-independent models of various forms were examined to determine their suitability for the purpose of predicting average annual individual tree basal area growth of natural even-aged stands of shortleaf pine throughout eastern Oklahoma and western Arkansas. A model was selected based on performance in both the study data and an independent data set. The resulting model performs well on average sites under normal stand densities. The model can be combined with mortality, DBH/height, and individual tree volume equations to predict the growth and yield of natural even-aged stands of shortleaf pine throughout eastern Oklahoma and western Arkansas. The study data did not include stands in which basal area growth had culminated, and few observations were available in young stands on high quality sites. For these reasons the model may not produce reliable results if applied under these conditions.

I would like to express my gratitude to the individuals who provided assistance in both the project and my course work. To be certain Dr. Thomas Lynch, my major adviser, provided guidance and invaluable advice during the entire ss. I would also like to thank the other members of my ttee Dr. Larry Gering and Dr. Mark Payton for their tance during the course of this project.

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CHAPTER 1

INTRODUCTION

The Shortleaf Resource

Shortleaf pine Pinus echinata is the most widely distributed of all the southern pines. It is found in 22 states, from Pennsylvania to Texas (McWilliams et. al., 1986). Shortleaf pine is second only to loblolly pine in softwood inventory volume for the South (Willet, 1986). In the Ouachita Highlands of Oklahoma and Arkansas shortleaf pine is the dominant softwood species, representing at least half of the softwood growing stock (van Hees, 1980). While public agencies and forest industry have large holdings, private individuals own a majority of the commercial timberland in the eastern Oklahoma and western Arkansas (Birdsey and May, 1988). Shortleaf pine is a major component of many of these small privately owned stands.

Shortleaf Management

Shortleaf stands are often regenerated naturally. This is because high site-preparation costs frequently make artificial regeneration less attractive to private land owners. In contrast, forest industry tends to replant loblolly pine Pinus taeda to achieve faster growth rates in short rotations. Although loblolly pine is considered to have a faster growth rate, shortleaf stands may tend to make up for this by maintaining more stems per acre (Maple and Mesavage, 1958) especially over longer rotations. Recent trends in management of public lands in eastern Oklahoma and western Arkansas are towards natural regeneration of existing stands. When silvicultural systems such as shelterwood and seedtree are used, the resulting stands will be even-aged. Because shortleaf pine is present in many of these stands and much of the area lies outside the natural range of loblolly pine (Harlow et. al., 1978), many of these stands will consist mostly of shortleaf pine.

Growth and Yield of Shortleaf Pine

Despite the importance of the shortleaf resource, relatively little is known about the growth of shortleaf pine in eastern Oklahoma and western Arkansas. Growth and yield models for managed stands of shortleaf pine are needed as they will provide the basis for comparisons of management alternatives. These models should incorporate variables which are easily measured or are included in a typical timber inventory. In addition, the models should be biologically reasonable. The Southern Forest Experiment

Station is currently planning such a growth and yield study for eastern Oklahoma and western Arkansas. Development of a basal area growth model is necessary as a component of this project. Data from permanent plots in the region are now available for the development of this model.

Objectives

The objectives of this study are:

1. To develop a distance-independent individual tree basal area growth model for natural even-aged stands of shortleaf pine in eastern Oklahoma and western Arkansas, which is sensitive to site, stand density, and individual tree characteristics. The model should be biologically reasonable, and should use common inventory measurements such as site index[1], age, stand basal area[2], and DBH[3].

2. To validate the model and make recommendations for its application in eastern Oklahoma and western Arkansas.

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1 Site index is a measure of site quality, and is calculated as the height in feet of dominant and codominant trees at age 50.

2 Stand basal area refers to the cross-sectional area (ft^2) at a height of 4.5 feet of all trees on an average acre in the stand.

3 DBH refers to the diameter of an individual tree at breast height (4.5 feet).

CHAPTER II

LITERATURE REVIEW

The methods used previously to model the growth and yield of shortleaf pine in the southern United States are related to an extensive body of work that includes many other forest types and regions. Williston (1975) compiled a bibliography of growth and yield models for southern pines which includes most work done before 1975. Dudek and Ek (1980) presented a bibliography of individual tree growth models worldwide. Trimble (1981) documented an inventory of growth and yield models for the United States. In addition Murphy (1986) presented a brief review of past work concerning the growth and yield of shortleaf pine. Readers should refer to these works for a complete reference of models developed before the 1980's. This review will concentrate on growth and yield models for shortleaf pine in the southern United States, and on models for other species and regions that pertain to the development of a basal area growth model for individual trees in natural even-aged stands of shortleaf pine.

Data Classification

A variety of data types have been used in the development of forest growth and yield models. To facilitate discussion of these data types, the classification system suggested by Moser and Hall (1969) will be used consistently below.

Real Growth Series

The ideal data for growth studies would be from stands maintained from regeneration to final harvest. This type of data would be termed a real growth series. Such data would provide insights into the dynamics of stand growth and development that would not be available from other data sources (Rogers and Sander, 1984). However, real growth series are very expensive to maintain and extremely long time periods are needed before the data are available.

Abstract Growth Series

Abstract growth series overcome some of the limitations of real growth series by substituting space for time. Temporary plots similar in all current stand characteristics except stand age are selected. Then plots within stands of different ages are compared to estimate growth. The advantage of abstract growth series is that long time periods are not required for compilation of data. This type of data is also less expensive, since there are no permanent plots to establish and maintain. The disadvantages of abstract growth series are that the past histories of the stands are not known and may be dissimilar, important treatment combinations may not be available, and these data are difficult to apply to individual tree growth model development since individual tree growth is not measured.

Approximated Real Growth Series

Several properties of approximated real growth series are intermediate to those of real growth series and abstract growth series. This type of data is collected from permanent plots established in existing stands and monitored through time. Since individual tree data are maintained, approximated real growth series data are useful for individual tree growth modeling. Other advantages of approximated real growth series are that approximated real growth series reflect the actual management of the stand, history of the stand is known (at least for the period of interest), and important characteristics can be controlled. There is, however, a time lag before data is available and permanent plots can be expensive to establish and maintain.

Model Classification

Many forest growth models can be classified as stand level stand-average models, stand level diameter class models, or individual tree models. Most forest growth and yield models that have been developed for shortleaf pine fall into one of these three categories.

Stand Level Stand-Average Models

The earliest work describing yields of shortleaf pine was done at the stand level. These models predict growth of the entire stand based on stand characteristics. Stand level models may be density-free or density-dependent.

Density-Free Stand Level Models are based on arbitrary stocking levels[4]. Therefore, they do not represent the growth of individual stands having other levels of stocking. Density free models based on the concept of full stocking do set an upper bound for the maximum growth one might expect for a given site and age. The earliest extensive work for shortleaf pine in the Southern United States was done by the USDA Forest Service (1929) and was published as

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4 A stocking level is a measure of the extent to which a stand is fully occupying the site.

Miscellaneous Publication No. 50. This study was based on data from an abstract growth series collected across the Southern Region. The growth represents the growth of "normal" or fully stocked stands, and is presented as a series of tables and graphs. Since the growth applies only to fully stocked stands, the growth of individual stands must be inferred by interpolation. Schumacher and Coile (1960) studied the growth of "well stocked" stands of southern pines. The data for this study came from an abstract growth series in the North Carolina Piedmont. Regression equations were presented to predict the change in stocking level, average height of the dominant stand, and number of trees per acre. Using the predicted stocking level and height of the dominant stand, future stand basal area can be inferred.

Variable-Density Stand Level Models predict growth as a function of site, age, and density. Stand density may be expressed as basal area per acre, trees per acre, or even volume per acre.

Murphy and Beltz (1981) developed a variable-density growth and yield model for shortleaf pine in the West Gulf Region. This was based on an approximated real growth series covering Oklahoma, Arkansas, Texas, and Louisiana. The data were obtained from USDA Forest Service permanent inventory plots in natural even-aged stands. Future stand basal area per acre was predicted as a function of density and age.

This estimate of future basal area was then used to project volume as a function of site index, age, and stand density. Murphy (1982) later developed models with the same data set to predict sawtimber volumes for the West Gulf Region. For stand level models in general, data requirements are minimal and can be obtained from limited inventory measurements. Unfortunately, many stand level stand-average models do not allow forest managers to segregate growth by product groups, such as pulpwood and sawtimber, within the model. Leary et. al. (1979) suggested this problem could be overcome by using a growth allocation rule. They suggested there would be a relation between the proportion of stand growth attributed to an individual tree and the proportion of stand growing stock attributed to the same tree. However, they found the relation to be nonlinear, extremely complicated to quantify, and quite variable depending on site conditions.

The projected stand basal area obtained from the model developed by Murphy and Beltz (1981) can be used as the independent variable in models developed by Lynch et. al. (1991) to predict merchantable cubic-foot volume, sawtimber cubic-foot volume, Doyle board-foot volume, Scribner board-foot volume, and International board-foot volume for shortleaf pine in the Ouachita Region of Oklahoma and Arkansas. Because the data for the stand basal area projection model were obtained from inventory plots, the model may not represent the growth of managed stands (Murphy, 1986).

However, like other stand level models, it does provide a basis for comparison of various sites and rotation ages.

Murphy et. al. (1992) presented the results of a study initiated in 1963 to investigate the effects of various measures of stand structure on periodic growth of natural even-aged shortleaf pine stands in the Ouachita Highlands. Data from an approximated growth series were used to develop equations to predict future stand basal area as a function of initial age, initial stand basal area, and age at prediction. Equations were also developed to predict total cubic-foot volume, sawtimber cubic-foot volume and various board-foot volumes, given initial age, initial standing volume, and age at prediction.

Stand Level Diameter Class Models

Diameter class models predict yields within diameter classes. These models can typically be classified as stand table projections or diameter distribution models.

Stand Table Projection Methods predict the growth of the average tree in each diameter class. The number of trees in each diameter class is then predicted, and growth for each class is obtained. In these models growth is determined by examining increment cores from sample trees within the stand. A basic assumption of many of these models is that a tree of a particular present diameter will grow the same as a tree which was the same diameter in the past. This assumption is not valid in even-aged stands, since a six-inch DBH tree in the present stand may be an intermediate while a six-inch DBH tree five years ago may have been a dominant or codominant (Clutter et al, 1983). In addition stand table projections often do not use site or density explicitly. In the past, stand table projection has often been the only alternative available for the prediction of yields by diameter class in natural shortleaf pine stands in the Ouachita Region of eastern Oklahoma and western Arkansas.

Diameter Distribution Methods are often used at the stand level. These methods model the distribution of trees in individual diameter classes. This is done using a continuous probability distribution function such as the Weibull distribution. Growth can then be inferred by examining the movement of trees to larger diameter classes. Much of the recent work in diameter distribution models has made use of the Weibull distribution. This is a three-parameter distribution whose cumulative distribution function, unlike the normal, exists in a closed form. Smalley and Bailey (1974) used the Weibull distribution to predict yield by diameter class in old field plantations of shortleaf pine. An abstract growth series obtained in Tennessee, Alabama, and the Georgia Highlands provided the data for this study. The Weibull parameters were predicted as functions of average stand height and number of trees surviving. These parameter estimates were then used to estimate diameter distributions for various combinations of site index (base age 25), age, and planting density. From these distributions and the number of trees surviving, stand tables[5] and stock tables[6] were generated. The procedure was found to predict diameter distributions as well as yield, quite satisfactorily (Smalley and Bailey, 1974).

Individual Tree Models

Individual tree models predict the growth of individual trees based on the characteristics of that particular tree and the forest stand in which it is located. The individual estimates are then summed to obtain stand estimates. Individual tree models are data intensive, since independent variables are needed for each tree. Processing time needed to predict future forest stand conditions is also increased, due to the large number of calculations required to predict future characteristics of each tree in the stand. Advances in computing technology have made individual tree models more practical in recent years, since most personal

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5 Stand tables present the number of trees in each DBH class for a particular stand.

6 Stock tables present the yield of a particular stand by DBH class.

computers are capable of preforming the necessary calculations. Because growth is predicted for each tree individually, these models are capable of segregating growth into various product or species groups. Individual tree models may be distance-dependent or distance-independent depending on how the competition index[7] is calculated.

Distance-Dependent Models use distances between individual trees to calculate a competition index for each tree. These models are computationally complex and require mapping of the entire stand, which can be quite expensive. To date, the only distance-dependent individual tree model applicable to one of the four major southern pines is the PTAEDA model developed by Daniels and Burkhart (1975). The PTAEDA model was developed for old-field loblolly pine plantations. The model consists of two stages. First, the initial stand is generated. To do this, positions of trees are generated as a function of the original planting density. Diameters are then assigned to individual trees using a two parameter Weibull distribution. The second stage simulates the growth, mortality, and competition of individual trees under various levels of thinning and fertilization. The growth of open grown trees is used as a measure of potential growth which is then reduced by a competition-based modifier. This

7 A competition index for an individual tree is any index that estimates the total competition from adjacent trees thought to be affecting the growth of the subject tree (Biging and Dobbertin, 1992).

growth is adjusted based on the particular management regime of the stand.

Daniels et. al. (1979) adjusted the initial stand generator portion of this model to examine seeded loblolly pine stands and found the model adequately described initial stand generation. However, the growth functions developed for plantations did not adequately describe the growth of seeded stands.

Burkhart et. al. (1987) revised the original PTAEDA model for applications in cut-over, site-prepared areas and found the new model adequately described stand development. While it would appear that distance-dependent models would be better able to quantify the competition experienced by individual trees, distance-dependent models have not proven to be clearly superior to distance-independent models (Davis and Johnson, 1987).

Distance-Independent Models use both individual tree and stand characteristics to estimate the level of competition affecting each tree. These models are not as computationaly complex and are less data demanding than distance-dependent models.

Two approaches have been used in distance-independent individual tree models. The first approach is to model tree growth directly as a function of individual tree and stand characteristics. The second approach has been to model tree potential growth based on individual tree characteristics. Then the potential growth is reduced by a modifier, which is a function of both tree and stand characteristics. These methods can be described as "direct models of stem growth" and "modified potential models of stem growth" respectively.

The direct model of stem growth method was used by Wykoff (1990) to model the growth of conifers in the Northern Rocky Mountains. The model was the same growth function used in the PROGNOSIS (Wykoff, 1986) growth and yield model developed for western conifers. An approximated real growth series conducted throughout the Inland Empire provided the data for this study. The model predicts the natural log of squared diameter increment as a function of tree size, site, and competition. Since site index is difficult to quantify in irregular mixed stands, site effects were modeled as a function of slope, aspect, elevation, habitat, and regional location. Competition was modeled as a function of crown ratio[8], basal area of all trees as large or larger than the subject tree, and crown competition factor as described by Krajicek et. al. (1961). The model was found to be adequate for describing the growth of individual trees in mixed species stands throughout the Inland Empire.

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8 Crown ratio is the ratio of the live crown to total height. It is generally felt that crown ratio provides a suitable index for the photosynthetic capacity of an individual tree.

Hilt and Dale (1982) used the direct model of stem growth approach to model the growth of even-aged upland oaks. Although the stand dynamics associated with the oak forest type are considerably different than those associated with shortleaf pine, the design of the model is unique and therefore warrants inclusion in this review. Three models were developed: a mean model, a random model, and a random/known model.

The mean model was developed in two stages. First, 5-year basal area growth was modeled as a function of squared DBH. Next the coefficients were estimated as a function of site index, quadratic mean diameter[9], and percent stocking. This procedure was followed for each growth period on each of the 77 plots included in the study.

In order to allow trees to change positions in the stand, a random model was developed. The prominent feature of the random model is that the 5-year growth of an individual tree is randomly selected from the distribution of the 5-year growth for a tree of the same size. In order to provide more realistic results, the mean growth of successive growth periods is related to the mean growth selected for previous growth periods. The predictions can be further improved by supplying the actual growth for the first iteration if it is known. The models were found to perform well.

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9 Quadratic mean diameter is a measure of central tendency. It is found by dividing the stand basal area per acre by the number of trees per acre, then calculating the diameter associated with this average basal area.

However, the models were tested using the same data set that was used to develop the model.

Quick et. al. (199_) used a combination of both methods to model the growth of longleaf pine. The model was of the form of the product of potential and modifier. Rather than fitting the potential and modifier functions in separate stages, the entire model was fitted to the data set simultaneously.

Murphy and Shelton (1993) also used this method to model the growth of loblolly pine in both even and uneven-aged stands. The confounding of potential and modifier effects noted by Wensel et. al. (1987) due to simultaneous fitting of both components was minimized by the choice of equation forms. The modifier used was the logistic function, which is bounded by the interval (0,1). The potential portion of the equation does not set a specific maximum tree size. However, it does approach zero as individual tree basal area approaches infinity. The equation was tested using approximated real growth series data from both even and uneven-aged stands, and was found to perform reasonably well in both cases. The model presented can be used to predict annual tree basal area growth in even-aged stands as a function of individual tree basal area, age, and stand basal area. Annual tree basal area growth in uneven-aged stands is modeled as a function of individual tree basal area, basal area of all trees as large or larger than the subject tree, site index, quadratic mean diameter, and stand basal area.

The modified potential stem growth method was used in the development of growth functions for STEMS for Lake States species (Belcher et. al., 1982). Hahn and Leary (1979) developed several potential growth functions for Lake States species. The data for this study were collected from an approximated real growth series conducted throughout the Lakes States Region. The potential function was fit by grouping growth by one-inch DBH classes, ten-foot site index classes, and ten percent crown ratio classes. For each group, average diameter growth and the standard deviation of average diameter growth was calculated. Potential growth for each group was taken to be the average growth of the group, plus 1.65 times the standard deviation of average growth. This potential growth corresponds to the 95th percentile of a normal distribution. A modifier function was then developed by Leary and Holdaway (1979) which was a function of size, site, and competitive status. As the stand basal area approaches the maximum basal area the modifier approaches zero. As competition experienced by the tree decreases, the modifier approaches one. Therefore, the growth of an individual tree is constrained to be greater than zero and less than the potential growth.

Shifley (1987) also used the potential growth concept in the development of a generalized system of models for forecasting tree growth in the Central States. An approximated real growth study from Missouri, Indiana, and Ohio was used to fit a modification of the TWIGS model (Belcher et. al., 1982) to estimate growth of individual trees. A Chapman-Richards function was used to describe the growth of the fastest growing 5% of trees in each class, since it could be constrained to a maximum tree size (Shifley and Brand, 1984). A competition function was added to the Chapman-Richards function to improve the estimate of potential growth. After parameters were estimated for the potential function, the modifier, which is a variation of the STEMS modifier, was included and the resulting equation was fit to the entire data set. The model was evaluated for several species with reasonable results. However, Shifley found that the model explained less than 31% of the variation in growth for shortleaf pine.

Wensel et. al. (1987) also used the modified potential stem growth method to model the squared diameter growth of Northern California conifers in both pure and mixed stands as a function of site index, age, diameter, height, live crown ratio, and trees per acre. An iterative procedure was used to estimate the model parameters. First, potential growth was estimated using a subset of the largest 33% of trees. Then competition modifier parameters were estimated using all the trees with the potential function paramters held constant. Finally, the competition coefficients were held constant and the potential function was refitted using all the trees in the data set. The resulting equations were incorporated into CACTOS, the California Conifer Timber Output Simulator (Wensel, Daugherty, and Meerschaert, 1986).

Bolton and Meldahl (1990) developed an individual tree distance independent multipurpose forest projection system for a variety of forest types, including shortleaf pine, found throughout the South. The data used in the development of this model were from an approximated growth series conducted throughout Georgia. The data for this model were collected as part of the USDA Forest Service Survey and therefore may suffer the limitations discussed earlier for this type of data. Regardless, the authors felt these limitations did not severely hamper the performance of the model, and that the model would prove useful in the absence of better data. Models were developed to predict live crown ratio, annual diameter increment, bole length, and mortality. The models were then incorporated into the TWIGS 2.0 framework (Belcher et. al., 1982). Since the model was to be used to simulate the growth of a wide variety of forest types, cluster analysis was used to group species which exhibited similar characteristics of interest. Multiple linear regression was used to model annual diameter increment because nonlinear models did not produce satisfactory results for the data. The fit and behavior of the model was improved using an iterative method of fitting a power transformation to annual diameter increment and then refitting the multiple linear regressions. This transformation procedure assumed that annual diameter increment was greater than zero (an assumption which is not necessarily compatible with the data of the current study). The authors suggest that the models will perform well on average. However, the model may behave illogically in individual situations.

Another approach to the modified potential growth method was used by Amatieus et. al. (1989). It was suggested that the growth of open grown trees would be a reasonable potential growth for modeling the growth of loblolly pine in both thinned and unthinned stands. This potential growth was then adjusted by a modifier, which was a function of crown ratio and relative size. The model was found to be useful for comparisons of various levels of thinning and hardwood control in loblolly plantations on cutover lands throughout the natural range of loblolly pine.

Smith et. al. (1992) studied the growth of open grown shortleaf pine in southeastern Arkansas. Basal area growth is given as a function of tree size. The equation was intended for use as potential growth in future growth models. Crown width of open grown trees was also modeled and this equation can be used for various forms of competition indices such as crown competition factor (Krajicek et. al., 1961).

Other Works

In 1955 a thinning study was begun for natural even-aged stands of shortleaf pine on the Sinkin Experimental Forest in Missouri. Five density levels were examined to determine the growth and yield each would produce. The results of this ten-year study were reported by Brinkman, Rogers, and Gingrich (1965). They found that maximum yield could be obtained when 30 year old stands were thinned to 70 square feet of basal area. Twenty-one-year results from the same study were presented by Sander and Rogers (1979). They found that repeated thinning to low basal areas (50 and 70 square feet per acre) left the stand understocked by age 51. The thirty-year results of the same study were also presented by Rogers and Sander (1984). They found that hardwoods significantly reduced the growth and yield of the shortleaf stands in this study.

The data for this study were obtained from a very small geographic region and a narrow range of site qualities. Despite these limitations the results have proven useful on similar sites in the same locale.

CHAPTER III

DATA

The data for this study were obtained from permanent plots located within the Ozark and Ouachita National Forests in eastern Oklahoma and western Arkansas. These plots were established from 1985-87 as part of USDA Forest Service Study 48 conducted cooperatively by the Southern Forest Experiment Station in Monticello, Arkansas and the Department of Forestry at Oklahoma State University. A detailed description of the data collection procedure is given by Murphy (1988).

Stand basal area, site index, and age were the basic independent variables used in the study design. Four levels of each basic independent variable were selected to cover the range of conditions found throughout the Ouachita Region (see Table I, page 24).

Plots were selected in areas meeting the following criteria:

- Natural stands with no obvious holes or clumping,
- Even-aged stands with a single canopy level and a maximum range in age of 10 years,
- Stands consisting of at least 70% shortleaf pine,

- Stands free of insect, disease, or fire damage, and
- Stands which had not been cut during the past five years.

TABLE I

VARIABLE COMBINATIONS USED IN THE STUDY DESIGN

Variable	Units	Class Range	Class Midpoint
Basal Area	Square Feet	16-45	30
		46-75	60
		76-105	90
		106-135	120
Site Index (base age 50)	Feet	<56	
		56-65	60
		66-75	70
		>75	
Age	Years	11-30	20
		31-50	40
		51-70	60
		71-90	80

Preliminary estimates of site index and age were based on height and increment core data from five dominant or codominant trees in each stand. Stand basal area was estimated using a 10 factor prism. These estimates were used to select prospective stands and assign them to a reasonable treatment combination. The original design specified three plots in each combination of the age, site index, and basal area classes. Only 2 plots were located for the treatment representing age of 20 years, stand basal area of 30 square feet, and site index >75 feet. Also seven plots were lost due to changing management schemes, damage, or failure to thin. This resulted in 183 plots being available for use. Estimates of site index and age made prior to plot establishment were not consistent with plot measurements in a number of cases. As a result, the number of plots established for young ages having site index greater than 65 feet was significantly less than anticipated. A validation data set containing 1964 observations was removed from the original data leaving 5592 observations to fit the various models. See Table II, page 27 for a summary of the data used for this study.

Three circular 0.2 acre plots were established in natural even-aged stands of shortleaf pine for each treatment combination. Each plot was surrounded by a 33 foot buffer in order to eliminate edge effects. Hardwoods were controlled with herbicide and the pine component was thinned to the desired treatment level on both the plot and buffer. All remaining shortleaf pine having DBH greater than or equal to one inch were permanently numbered and DBH and crown class[10] were recorded for each.

On each plot a subset of trees was selected to represent each diameter class in proportion to the number of

10 Crown class refers to the standing of the tree within the stand and that trees ability to compete for a position in the main canopy.

trees in the class. For each tree in this subset total height and height to live crown were recorded.

Age was also recorded for the dominant and codominant trees in the subset. Age was determined from ring counts on increment cores taken at breast height. The age of each tree was calculated as the number of annual rings plus five.

Site index (base age 50) was calculated using total height, age and an algorithm based on the site index equations presented by Graney and Burkhart (1973) for shortleaf pine in the Ouachita Region. Stand basal area was calculated by summing individual tree basal areas and expanding to a per acre basis. Crown ratio was calculated as the ratio of crown length to total height. Basal area of all trees as large or larger than the subject tree (BAL) was also calculated and transformed to a per acre basis. Other variables required to fit prospective individual tree basal area growth models were obtained when needed. Finally, whenever appropriate, midpoint values of the independent variables were used. Midpoint values were calculated as the average of the first and second measurements. In cases where a tree failed to survive for the entire measurement period, half the initial measurement value was used as described by Bolton and Meldahl (1990).

TABLE II

SUMMARY OF DATA AVAILABLE FOR THE DEVELOPMENT OF AN INDIVIDUAL TREE BASAL AREA GROWTH MODEL

Variable	Minimum	Maximum	Mean	Standard Deviation
Age	20	96	42.1	19.0344
Site Index	37	87	55.1	9.9612
Stand Basal Area	22.5	142	102.4	30.3007
DBH	1.2	24.9	7.52	3.8086
Average Plot Crown Ratio	0.2627	0.5882	0.3919	0.0527
Crown Competition Factor	16.1	292.7	160.6	60.6338
Average Annual Ind. Tree Basal Area Growth	-0.0603	0.0718	0.0130	0.0106
Ind. Tree Basal Area	0.0072	3.3830	0.3878	0.3884
Basal Area of all Trees as Large or Larger Than the Subject Tree	0	141.29	64.10	37.08

7556 observations.

CHAPTER IV

METHODS

In order to develop an individual tree growth model for natural even-aged stands of shortleaf pine, models used in previous studies were examined. The models were screened based on the following criteria:

- the model must be biologically reasonable,
- the dependent variable should be diameter growth, basal area growth, or some function of either,
- the independent variables should be those that are available from most forest inventories, and
- the model should fit the data.

Selection Criteria

Biologically Reasonable

Because the model may be applied outside the geographic range of data as well as to stands with characteristics not found in the data, the model should be biologically reasonable. Such a model must reach a realistic maximum tree size under normal stand conditions, and should follow a logical growth pattern. The model should respond logically to changes in the independent variables. Finally, the model should continue to perform logically outside the range of this study data.

Dependent Variables

The growth of individual tree basal area may be modeled directly or it may be obtained from DBH growth, since basal area = $k \cdot (DBH)^2$, where k is a constant that depends on the units of measurement. Various transformations of DBH or basal area growth may also be considered for use as the dependent variable. The choice of dependent variables has received a great deal of attention in the past. West (1979) found that basal area increment was more highly correlated with the independent variables used to predict growth than diameter increment. However, he suggested that this higher correlation was due to the fact that basal area increment was dependent on initial tree size. He went on to show, for several species, that there was no significant difference in the precision of estimates of future tree size provided by use of either DBH growth or basal area growth as a dependent variable in growth models. Shifley (1987) also found that there was little evidence to favor use of either DBH growth or basal area growth as a dependent variable in terms of prediction accuracy. Hilt and Dale (1982) suggests the relationship between tree growth and tree size is graphically more distinct when basal area is used as the dependent variable. Also, individual tree volumes are usually proportioned to the product of tree height and basal area. Therefore, Shifley (1987) suggests that it would be logical to predict basal area growth directly. Arguing in favor of using DBH growth as a dependent variable is the fact that diameter growth culminates before basal area growth in most stands so that diameter growth may be more easily constrained for young stands. It appears the choice of dependent variables depends largely on the objectives of each individual study and the preferences of particular investigators, because when initial diameter is known either measure can be easily calculated. A final consideration is the fact that fit statistics for basal area growth are often much higher than those for diameter growth in the same stands (Shifley, 1987). This is because the range of diameter growth is smaller, relative to the mean diameter growth, than the range of basal area growth to mean basal area growth. Thus, the corrected total sum of squares, which is the denominator of the fit indices, is smaller for DBH growth than for basal area growth. It is necessary to account for this when comparing models with different dependent variables. In this study individual tree basal area growth was selected as the primary dependent variable because of its more direct relationship to volume and stand basal area, its successful use in many previous studies, and its acceptable performance with the current data set.

Periodic individual tree basal area growth was used to estimate average annual basal area growth. This estimation was necessary for two reasons. First, a standardized time period for growth was required since growth was measured for both four- and five-year periods. Also, annual growth estimations will allow the model to be used for various projection periods.

Independent Variables

The practical utility of a basal area growth model is enhanced when the independent variables used in the model are available from most forest inventories. Because it is difficult to characterize the spatial distribution of individual trees accurately in a natural stand, the distance-independent approach to individual tree modeling was selected. Also measures of competition used in the model should be available or computable from typical forest inventory data. Forest inventory variables which are often measured directly include site index, stand age, diameter of individual trees, and stand density. From these basic measurements it is possible to compute various measures of competition such as basal area of all trees as large as or larger than the subject tree, crown competition factor (Krajicek et. al., 1961), and average plot crown ratio.

Fit of the Model

The fit of the model to the data is important for the comparison of model performance. The fit index described by Shifley (1987) was used for preliminary evaluation of fit. This is a measure of the proportion of variation explained by the model. The fit index formula is:

$$1-\left\{\frac{\text{ERROR SUM OF SQUARES}}{\text{CORRECTED TOTAL SUM OF SQUARES}}\right\}$$

Various statistics which are described below were then used to evaluate the performance of selected models on an independent data set. Average Deviations were calculated as the sum of the differences of predicted average annual basal area growth and actual average annual basal area growth, divided by the number of observations. Mean Square Errors were calculated as the sum of the squared differences of predicted average annual basal area growth and actual average annual basal area growth, divided by the number of observations. Mean Absolute Deviations were calculated as the sum of the absolute differences of predicted average annual basal area growth and actual average annual basal area growth, divided by the number of observations. Average Percent Errors were calculated as the product of 100 and the sum of the differences of predicted average annual basal area growth and actual average annual basal area growth, divided by the product of actual average annual basal area growth and the number of observations. Graphical analysis of the residuals vs. independent variables were also examined to check for bias.

Models Examined

The modeling of growth of individual trees can be approached in two ways. The first approach is to predict growth directly as a function of individual tree and stand characteristics. The second approach is to use the potential growth of a tree growing free of competition, the proportion of this potential growth actually realized is then modeled. Martin and Ek (1984) classified these methods as "empirical" and "semi-empirical" respectively. Because the former class of models can include theoretical aspects, the classes "direct models of average stem growth" and "modified potential stem growth" are used below.

Direct Models of Average Stem Growth

These models attempt to explain variations in tree growth by modeling deviations about the mean growth of trees. One of the more recent, and most widely used of these models is the growth model of PROGNOSIS (Wykoff, 1986). PROGNOSIS is an individual tree growth and yield model for the Inland Empire of the Northern Rocky Mountains. It was developed by the USDA Forest Service for young managed stands. The growth model at the heart of PROGNOSIS predicts the natural log of basal area increment using site, habitat, and individual tree characteristics. A detailed description of the model formulation is given by Wykoff (1990). Briefly the model is based on an equation using the natural log of diameter and diameter squared as independent variables. Site and competitive intercepts are added to further explain variations in growth. Since the model covers a wide geographic range with stands of mixed species and ages, variables were used to quantify location and site effects. For application to the data used in this study the location effects are replaced by a single intercept and the site effects are estimated by site index. Because the model will be applied to even-aged stands a term was added to represent the age of the stand. The resulting model is:

$$\ln(bai) = \beta_0 + \beta_1 \cdot \ln(D) + \beta_2 \cdot D^2 + \beta_3 \cdot BAL + \beta_4 \cdot CR + \beta_5 \cdot CR^2 + \beta_6 \cdot CCF + \beta_7 \cdot SI + \beta_8 \cdot AGE \qquad (1)$$

where:

bai = average annual tree basal area increment,
D = tree diameter at breast height,
BAL = total basal area of all trees in stand as large or larger than subject tree,
CR = average crown ratio of all trees in stand,
CCF = crown competition factor (Krajicek et. al., 1961),
SI = stand site index (base age 50),
AGE = stand age, and
β_i = regression coefficients to be estimated.

The logarithmic transformation of this model was fitted to the data using the REG procedure in SAS (SAS Institute Inc, 1989). The model was not weighted since the log transform of basal area increment removed any heterogeneity of variance. The model provided an R^2 value of 0.744 and mean square error of 0.26303. When transformed to predict average annual basal area growth directly the model provided a reasonable fit to the data set with a fit index of 0.593 and mean square error of 0.000046. The parameter values associated with the independent variables are reasonable since growth increases with an increase in crown ratio and site index, and growth decreases with increases in age and competition (Table III, page 36). Under average stand conditions maximum tree growth occurs at 21 inches DBH and maximum tree size is greater than 70 inches DBH. The maximum tree size is too high for naturally occurring shortleaf pine in western Arkansas and eastern Oklahoma, since the National Register of Big Trees reports the largest shortleaf pine as less than 43 inches DBH (American Forestry Association, 1992) and historical records indicate that trees in excess of 30 inches DBH are rare in the study Region (Smith, 1986). The excessive maximum tree size may be due to a large number of small diameter trees in the data and the fact that the data set does not include stands in which basal area growth has culminated.

TABLE III

PARAMETER VALUES FOR EQUATION #1 WHEN FITTED TO THE STUDY DATA

Parameter	Parameter Estimate	Standard Error
β_1	-9.029695	0.2962697
β_2	1.951544	0.0404550
β_3	-0.002458	0.0002579
β_4	-0.005681	0.0004237
β_5	7.327008	1.2628598
β_6	-6.156645	1.4916659
β_7	-0.001388	0.0001703
β_8	0.004683	0.0012538
β_9	-0.018637	0.0009368

Modified Potential Stem Growth

These models have dominated the literature in recent years. These models are useful because they set an upper limit on growth that can be achieved by a given tree (Hahn and Leary, 1979). Models of this type also divide the task of modeling into two more manageable parts (Shifley, 1987). The form of this model is theoretically logical in that trees are assumed to reach some proportion of potential growth based on the particular stand and site conditions. These properties are especially desirable for models intended for use in regional predictions where several species are involved, since the data sets are often large and the model form must apply to species which may have very different growth habits. Certain regional growth models developed by the USDA Forest Service such as TWIGS and STEMS have used this approach. Such models have the following form

actual growth = {potential}·{modifier}.

Several potential and modifier functions were considered for this study and a brief discussion of the theory and preliminary results follow.

Potential Functions are used to estimate the potential growth of a given tree. Several approaches to tree potential growth estimation have been explored. Hahn and Leary (1979) suggested the potential growth of trees could be estimated by using the 95th percentile of growth. The potential function developed by Hahn and Leary (1979) for Lake States species has the following form:

$$\text{Potential} = \beta_1 + \beta_2 \cdot D^{\beta_3} + \beta_4 \cdot SI \cdot CR \cdot D^{\beta_5} \qquad (2)$$

Where :

D = Tree DBH at midpoint
SI = plot site index (base age 50)
CR = average plot crown ratio, and
β_i = parameters to be estimated.

This potential function was fitted to the data set as described by Hahn and Leary (1979) with slight variations in the method used to separate trees into cells. The data were divided into cells by 2-inch DBH class, site index class (as described in the treatment combinations), and 20% crown

The equation was fitted to the fastest growing 5% of dominants and codominants in each 1 inch DBH class. If less than 20 observations were available for a particular class, all observations were included. The latter condition applied only to the largest DBH classes and therefore should be valid since these observations represent trees which were all growing at very nearly the same rate. The use of the fastest growing trees was again intended to predict the growth of trees which were free from significant competition. The model was fitted using the secant method of nonlinear regression in SAS (SAS Institute Inc., 1989). The Chapman-Richards function is especially suited for use as a potential function because it places an absolute limit on maximum tree size. The maximum size is attained when growth equals zero, and is given by the formula

$$\text{Max Basal Area} = (\beta_1/\beta_3)^{1/(1-\beta_2)}. \tag{4}$$

The maximum tree basal area for shortleaf pine data used in this study was 3.39 ft^2. This corresponds to a tree of approximately 25 inches DBH. This estimate was not considered to be large enough for the region of interest based on existing literature and experience.

The maximum tree size can be set to an arbitrary level by solving equation 4 for β_3 and replacing β_3 in equation 3 (Shifley and Brand, 1984). This yields the equation

$$Pot = \beta_1 \cdot B^{\beta_2} - \beta_1 / M^{(1-\beta_2)} \cdot B. \qquad (5)$$

Where:

Pot = potential basal area growth (ft^2),
B = individual tree basal area (ft^2),
M = maximum tree basal area (ft^2), and
βi = parameters to be estimated.

Historical records indicate that trees in excess of 30 inches DBH were rare throughout the Ouachita Region (Smith, 1986). The National Register of Big Trees lists a shortleaf pine of approximately 42 inches DBH as the largest on record (American Forestry Association, 1992). Based on this information, 36 inches was chosen as a reasonable estimate of the maximum size for shortleaf pine in natural stands of western Arkansas and eastern Oklahoma. Maximum size, or M, was set to 7.068384 and equation 5 was fitted to the fastest growing 5% of trees using the Marquardt method of nonlinear regression in SAS (SAS Institute Inc., 1989). The regression explained over 78% of the variation in growth for the fastest growing 5% of trees with a mean square error of 0.0000546 (see table IV, page 41). Shifley (1987) recommended the addition of a term for site index and crown ratio. However, these variables did not improve the ability of the equation to predict potential growth for this data set.

TABLE IV

PARAMETER VALUES FOR EQUATION #5 WHEN FITTED TO THE CALIBRATION DATA

Parameter	Parameter Estimate	Standard Error
β_1	0.078904	0.003089
β_2	0.561268	0.021300

Amateis and others (1989) suggested that the growth of open grown trees should be a reasonable approximation of the growth of forest trees which are free from competition. Smith and others (1992) studied the growth of open grown shortleaf pine in the West Gulf Region with the intent that the equation developed could be used as a possible potential function in growth models for shortleaf pine. The equation used was a variation of the Chapman-Richards function in which an intercept term was added. When growth data obtained from open-grown shortleaf pine trees were fitted to this modified Chapman-Richards equation form, the following was obtained:

$$\begin{aligned} PAIBBAG = {} & 0.00031 + 0.03165 \cdot IIBBA^{0.46922} \\ & - 0.03809 \cdot IIBBA. \end{aligned} \tag{6}$$

Where

PAIBBAG = potential inside bark basal area growth meters^2), and

IIBBA = initial inside bark basal area (meters^2).

Equation 6 uses basal area inside bark as the independent variable and is therefore not directly applicable using standard inventory measurements. Smith and others (1992) developed the following equation for estimation of double bark thickness.

$$DB = 0.75631 + 0.08879 \cdot DBH. \qquad (7)$$

Where

DB = double bark thickness (cm), and
DBH = diameter at breast height (cm).

Equations 6 and 7 can be used to develop the following algorithm to predict potential basal area growth (ft^2) as function of DBH (inches):

1. Calculate current inside bark basal area:

$$CIBBA = 0.000421 \cdot DBH^2 - 0.000275 \cdot DBH + 0.000045 \qquad (8)$$

Where

CIBBA = current inside bark basal area (meters²),
DBH = diameter at breast height (inches).

2. Calculate future inside bark basal area:

$$FIBBA = 0.00031 + (0.03165 \cdot CIBBA^{0.46922}) + 0.96191 \cdot CIBBA \qquad (9)$$

Where

FIBBA = future inside bark basal area (meters²),
CIBBA = current inside bark basal area (meters²),

3. Calculate potential annual basal area growth:

$$PABAG = 0.000582 + 0.17378 \cdot FIBBA^{0.5} + 12.96343 \cdot FIBBA - B \qquad (10)$$

Where

PABAG = potential annual basal area growth (ft^2),
FIBBA = future inside bark basal area ($meters^2$), and
B = current tree basal area (ft^2).

Because the potential function equals zero when a tree reaches a maximum tree size of approximately 42 inches, the modifier does not affect the maximum size.

Modifier Functions are used to explain variations from the potential growth predicted for each individual tree. Modifier functions are generally functions of site and stand characteristics as well as measures of competition. Several modifier functions were examined and two were selected for further analysis on the basis of their simplicity and initial performance.

The modifier function proposed by Shifley (1987) is a variation of the STEMS and TWIGS modifiers. This modifier adjusts potential growth according to the competitive status of the individual tree (as expressed by BAL) as well as the density of the entire stand (as measured by basal area per acre). The equation is

$$\text{Mod} = \beta_3 \cdot \{1 - \exp[-(\beta_4/(\text{BAL}+1) + \beta_5 \cdot \text{B}) \cdot (1 - \text{BA}/\text{BA}_{max})^{\frac{1}{2}}]\}. \quad (11)$$

Where:

Mod = proportion of potential growth actually achieved,
BAL = basal area per acre of all trees in the stand as large or larger than the individual tree (ft^2/acre),
B = basal area of individual tree (ft^2),
BA = stand basal area per acre (ft^2/acre),
BA_{max} = maximum stand basal area (ft^2/acre), and
β_i = parameters to be estimated.

The variable BA_{max} was set to 200 as recommended by Shifley (1987). This is probably a generous level, because the maximum basal area Miscellaneous Publication No. 50 (USDA Forest Service, 1929) reports is 174 ft^2 per acre for fully stocked stands of shortleaf pine. Equation 11 has a minimum value of zero when stand basal area reaches 200 ft^2/acre. The maximum value of equation 11 is equal to β_3, and is approached asymptotically as BAL and basal area per acre decreases to zero, and tree basal area increases. The modifier does not affect the maximum tree size established in equations 5 or 8, because these equations are equal to zero at maximum tree size.

A second modifier was proposed by Murphy and Shelton (1993) for use in both even and uneven-aged stands of loblolly pine. The modifier adjusts potential growth by an equation which is easily adapted to include a wide variety of stand and individual tree conditions. The form of the modifier is

$$\text{Modifier} = 1/\{1 + \exp[\beta_1 \cdot BAL + \beta_2 \cdot SI + \beta_3 \cdot AGE + \beta_4 \cdot BA + \ldots]\}. \quad (12)$$

Where

BAL = basal area of all trees as large or larger than the subject tree (ft^2),
SI = plot site index (base age 50),
AGE = plot age (years),
BA = stand basal area (ft^2), and
β_i = parameters to be estimated.

Additional variables can be added to the equation as they are needed to explain variations in growth. Equation 12 has a maximum value of 1, which is approached asymptotically as the exponential term approaches negative infinity. The equation's minimum value of zero is approached as the exponential term approaches infinity. If the potential growth value is known, the parameters of equation 12 can be estimated using either linear or nonlinear regression. The model can be fit using linear regression by rearranging the equation

$$BAG = PABAG \,/\, \{1+\exp[\beta_1 \cdot BAL + \beta_2 \cdot SI + \ldots]\} \quad (13)$$

as

$$\ln[(PABAG/BAG)-1] = (\beta_1 \cdot BAL + \beta_2 \cdot SI + \ldots). \quad (14)$$

Models Used

Combinations of the various potential and modifier functions as well as a loglinear model based on the PROGNOSIS model described by Wykoff (1990) were selected for evaluation in this study. The following models were selected for further examination based on preliminary results and the nature of model structures. See Table V, page 50 for the parameter estimates and fit indices obtained when these models were fitted to the study data.

Model 1 was a variation of the model used by Shifley (1987) for Central States trees. Equation 5 was used as the potential function, and equation 11 was used as the modifier. This yields the following model:

$$AABAG = \beta_1 \cdot B^{\beta_2} - \beta_1/M^{(1-\beta_2)} \cdot B \cdot$$
$$\beta_3 \cdot \{1-\exp[-(\beta_4/(BAL+1) + \beta_5 \cdot B) \cdot (1-BA/BA_{max})^{\frac{1}{2}}]\}$$

Where

AABAG = Average annual individual tree basal area growth (ft^2),
B, M, BAL, BA, BA_{max}, and βi are as previously defined.

Model 2 used equation 5 as the potential function, and equation 12 as the modifier. This yields the following model:

$$AABAG = \frac{\beta_1 \cdot B^{\beta_2} - \beta_1/M^{(1-\beta_2)} \cdot B}{\{1 + \exp[\beta_3 + \beta_4 \cdot BA + \beta_5 \cdot AGE + \beta_6 \cdot BAL]\}}$$

Where

B, M, BA, SI, AGE, BAL, and β_i are as previously defined.

The model was found to be biased with respect to diameter, therefore the potential function was refitted to the entire data while holding the modifier coefficients constant, as described by Wensel (1987). This procedure appeared to remove the bias with respect to diameter, and improved the behavior of the model in general, so no further iterations were performed.

Model 3 used equation 9 as the potential function and equation 12 as the modifier. This yields the following equation:

$$AABAG = \frac{POT}{\{1 + \exp[\beta_3 \cdot BA + \beta_4 \cdot SI + \beta_5 \cdot AGE + \beta_6 \cdot BAL]\}}$$

Where

AABAG = Average annual individual tree basal area growth (ft^2),
POT = Growth of open grown tree(Smith et al, 1992),
BA, SI, AGE, BAL, and β_i are as previously defined.

Model 4 was a variation of the PROGNOSIS model (Wykoff, 1990) and used equation 1.

$$\ln(bai) = \beta_1 + \beta_2 \cdot \ln(D) + \beta_3 \cdot D^2 + \beta_4 \cdot BAL + \beta_5 \cdot CR + \beta_6 \cdot CR^2 + \beta_7 \cdot CCF + \beta_8 \cdot SI + \beta_9 \cdot AGE$$

where:

AABAG= average annual individual tree basal area growth (ft^2),
bai = average annual tree basal area increment,
D = tree diameter at breast height,
BAL = total basal area of all trees in stand as large or larger than subject tree,
CR = average crown ratio of all trees in stand,
CCF = crown competition factor,
SI = stand site index (base age 50),
AGE = stand age, and
β_i = regression coefficients to be estimated.

TABLE V

PARAMETER ESTIMATES AND FIT INDICES FOR MODELS FITTED TO THE CALIBRATION DATA

Parameter	Model #1	Standard Deviation	Model #2	Standard Deviation	Model #3	Standard Deviation	Model #4	Standard Deviation
β_1	0.078904	0.00309	0.093961	0.00269	-	-	-9.029695	0.29627
β_2	0.561268	0.02130	0.643337	0.01206	-	-	1.951544	0.04046
β_3	0.648152	0.00620	-1.944122	0.06311	0.006103	0.00026	-0.002458	0.00026
β_4	87.260650	3.26923	0.007864	0.00050	-0.007561	0.00054	-0.005681	0.00042
β_5	0.137430	0.06948	0.010833	0.00067	0.010058	0.00046	7.327008	1.26286
β_6	-	-	0.014512	0.00047	0.011196	0.00032	-6.156645	1.49167
β_7	-	-	-	-	-	-	-0.001388	0.00017
β_8	-	-	-	-	-	-	0.004683	0.00125
β_9	-	-	-	-	-	-	-0.018637	0.00094
Fit Index	0.5623		0.5995		0.5962		0.5930	
MSE	0.000050		0.000045		0.000047		0.000046	

CHAPTER V

RESULTS

Models were evaluated using an independent data set of 1,964 observations. The predicted values were calculated for each model using the parameters estimated with the original data. The average deviations, mean square errors, mean absolute deviations, and average percent errors were calculated using these predicted values and the actual observed values. The statistics for each model were then compared to evaluate model performance. The residuals were also plotted against DBH, basal area per acre, age, and site index to identify any bias which might be associated with the models. The residuals were calculated as the predicted minus the actual value therefore, negative residuals indicate under prediction and positive values indicate over prediction. In order to more clearly illustrate trends in the residuals, box plots were chosen instead of traditional scatter plots. For a description of the meaning of the box plots used, please see figure 1, page 50.

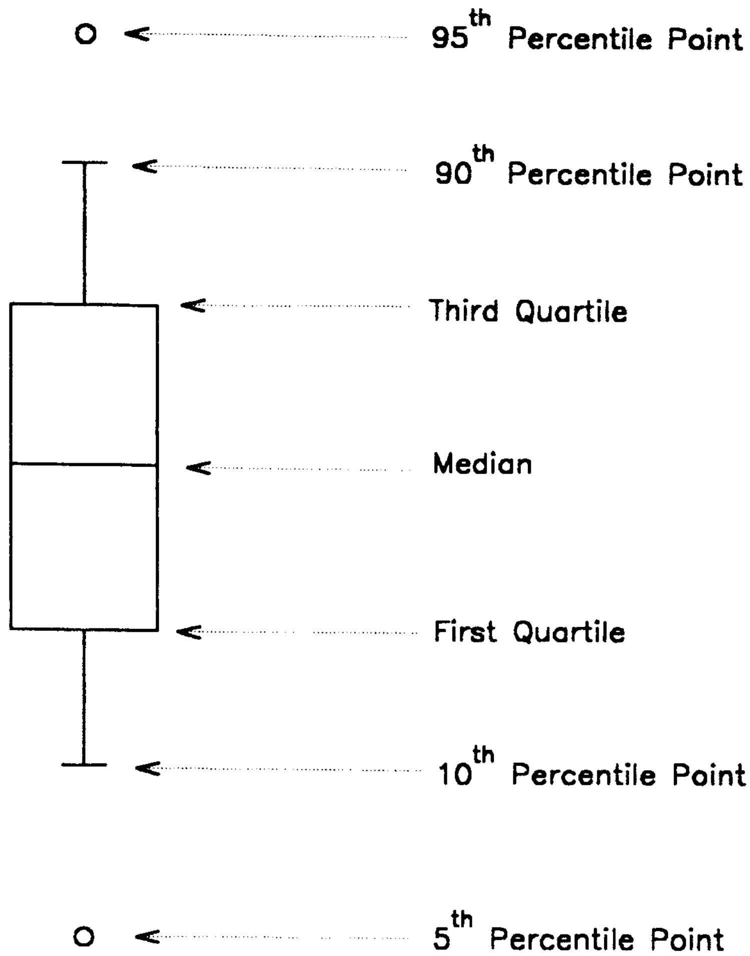

Figure 1. Description of Box Plots Used in the Residual Plots for Models Evaluated Using the Independent Data.

<u>Average Deviations</u> for each model were calculated by 2 inch diameter classes (see Table VI, page 51). Model 4 had average deviations which indicate the model tended to under predict growth across the range of DBH classes. Model 2 had the lowest average deviations across the entire range of data.

TABLE VI

AVERAGE DEVIATIONS OF MODELS BY DBH CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

Class	Obs	Model 1	Model 2	Model 3	Model 4
2	83	0.000361	0.000761	0.001958	-0.000970
4	566	-0.000451	0.000035	0.001130	-0.001492
6	336	-0.000736	-0.000086	0.000703	-0.000805
8	297	0.000421	0.000316	0.000702	0.000138
10	248	0.000135	-0.000595	-0.000626	-0.000540
12	208	0.003610	0.001507	0.000457	0.000198
14	130	0.003837	0.001160	-0.000426	-0.000560
16	51	0.002665	-0.000333	-0.001468	-0.001292
18	45	-0.003188	-0.005539	-0.005395	-0.005716
ALL	1,964	0.000472	0.000101	0.000414	-0.000836

Average deviations were also calculated by site index class, stand density level, and age class (see Table VII, page 52). Based on this criterion it would appear that model #3 has relatively large positive average deviations. This would indicate that the model may be biased with respect to DBH.

TABLE VII

AVERAGE DEVIATIONS OF MODELS BY SITE INDEX, BASAL AREA PER ACRE, AND AGE CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
<56	1115	0.000861	0.000713	0.000914	-0.001238
60	365	-0.000028	-0.000708	-0.000717	-0.000878
70	358	0.000663	-0.000097	0.000491	0.000058
>75	126	-0.002060	-0.002409	-0.000954	0.000290

BA/ac	Obs	Model 1	Model 2	Model 3	Model 4
30	259	-0.004185	-0.002567	-0.000957	-0.003057
60	420	-0.000132	-0.000070	0.000466	-0.002310
90	504	0.001768	0.001162	0.000997	0.000170
120	781	0.001505	0.000392	0.000465	0.000042

Age	Obs	Model 1	Model 2	Model 3	Model 4
20	742	-0.000936	-0.000299	0.000958	-0.001245
40	656	-0.000242	0.000175	0.000875	0.000242
60	334	0.002255	0.000209	-0.000710	-0.000404
80	232	0.004429	0.001013	-0.001010	-0.003203

Mean Square Error was calculated for each model by 2 inch DBH classes (see Table VIII). Model 2 had the lowest mean square error over the entire data set, though models 3 and 4 performed similarly well with respect to this criterion.

TABLE VIII

MEAN SQUARE ERROR OF MODELS BY DBH CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

DBH Class	Obs	Model 1	Model 2	Model 3	Model 4
2	83	0.000002	0.000002	0.000006	0.000003
4	566	0.000007	0.000006	0.000008	0.000010
6	336	0.000021	0.000018	0.000019	0.000018
8	297	0.000042	0.000038	0.000039	0.000042
10	248	0.000072	0.000064	0.000063	0.000065
12	208	0.000084	0.000064	0.000064	0.000066
14	130	0.000135	0.000098	0.000087	0.000079
16	51	0.000178	0.000151	0.000147	0.000127
18	45	0.000194	0.000233	0.000233	0.000264
ALL	1,964	0.000041	0.000041	0.000041	0.000042

Mean squared error was also calculated by site index, stand density, and age class (see Table IX). All models seemed to perform comparably based on this criteria.

TABLE IX

MEAN SQUARE ERROR OF MODELS BY SITE INDEX, BASAL AREA PER ACRE, AND AGE CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
<56	1115	0.000032	0.000025	0.000025	0.000026
60	365	0.000057	0.000048	0.000050	0.000054
70	358	0.000066	0.000057	0.000058	0.000056
>75	126	0.000117	0.000121	0.000119	0.000110

BA/ac	Obs	Model 1	Model 2	Model 3	Model 4
30	259	0.000079	0.000060	0.000051	0.000053
60	420	0.000050	0.000045	0.000047	0.000050
90	504	0.000051	0.000047	0.000050	0.000051
120	781	0.000036	0.000029	0.000030	0.000029

Age	Obs	Model 1	Model 2	Model 3	Model 4
20	742	0.000012	0.000009	0.000011	0.000014
40	656	0.000052	0.000047	0.000047	0.000045
60	334	0.000086	0.000076	0.000075	0.000067
80	232	0.000100	0.000078	0.000075	0.000089

Mean Absolute Deviations were calculated for each model by 2 inch DBH classes (see Table X), site index, stand density, and age (see Table XI, page 56). All models performed similarly well based on this criterion.

TABLE X

MEAN ABSOLUTE DEVIATION OF MODELS BY DBH CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

DBH Class	Obs	Model 1	Model 2	Model 3	Model 4
2	83	0.001239	0.001230	0.002091	0.001453
4	566	0.002184	0.001987	0.002388	0.002515
6	336	0.003611	0.003336	0.003363	0.003264
8	297	0.005209	0.004927	0.005027	0.005053
10	248	0.006341	0.006001	0.005930	0.005931
12	208	0.007260	0.006208	0.006271	0.006412
14	130	0.009301	0.007857	0.007330	0.006962
16	51	0.010340	0.009392	0.009185	0.008789
18	45	0.011070	0.012171	0.012392	0.012743
ALL	1,964	0.004795	0.004399	0.004533	0.004518

TABLE XI

MEAN ABSOLUTE DEVIATION OF MODELS BY SITE INDEX, BASAL AREA PER ACRE, AND AGE CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
<56	1115	0.003883	0.003459	0.003596	0.003716
60	365	0.005488	0.004954	0.005043	0.005137
70	358	0.005920	0.005595	0.005788	0.005137
>75	126	0.007659	0.007706	0.007783	0.007254

BA/ac	Obs	Model 1	Model 2	Model 3	Model 4
30	259	0.006386	0.005265	0.004691	0.005624
60	420	0.004604	0.004337	0.004708	0.005624
90	504	0.005124	0.004959	0.005103	0.005044
120	781	0.004150	0.003783	0.004018	0.003726

Age	Obs	Model 1	Model 2	Model 3	Model 4
20	742	0.002580	0.002257	0.002550	0.002798
40	656	0.005111	0.004945	0.005064	0.004804
60	334	0.007034	0.006522	0.006486	0.006101
80	232	0.007761	0.006647	0.006563	0.006929

Average Percent Error was calculated for each model by 2 inch DBH class (see Table XII, page 58), site index, stand density, and age classes (see Table XIII, page 59). The percent error was high for model 3 at the smaller DBH classes as well as low site indices and at young ages. Model 4 had the lowest average percent errors, with the other models all having considerably higher values. Average percent error is most affected by deviations in trees with growth approaching zero due to the influence of the denominator, therefore this criterion places more emphasis on those trees which are accumulating growth at lower rates. Since the data set contains no mature trees which would be approaching zero growth in the absence of competition, it is likely that deviations between predicted and actual growth of suppressed trees are most affecting this measure.

TABLE XII

AVERAGE PERCENT ERROR OF MODELS BY DBH CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

DBH Class	Obs	Model 1	Model 2	Model 3	Model 4
2	83	41.367	50.392	99.526	-11.713
4	566	26.081	31.236	58.749	4.545
6	336	11.008	16.718	33.150	12.907
8	297	31.102	25.385	29.428	23.209
10	248	24.265	14.309	14.035	12.202
12	208	51.190	32.706	23.750	20.647
14	130	45.961	26.749	16.150	14.023
16	51	7.939	2.877	8.882	7.749
18	45	22.696	11.437	9.729	8.406
ALL	1,964	28.105	25.059	37.068	11.582

TABLE XIII

AVERAGE PERCENT ERROR OF MODELS BY SITE INDEX, BASAL AREA PER ACRE, AND AGE CLASS WHEN EVALUATED USING THE INDEPENDENT DATA

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
<56	1115	28.731	27.859	38.915	2.782
60	365	32.365	25.143	33.559	20.358
70	358	26.416	20.460	39.874	25.400
>75	126	15.025	13.099	22.925	24.778

BA/ac	Obs	Model 1	Model 2	Model 3	Model 4
30	259	-12.814	- 4.846	4.159	-13.738
60	420	12.208	16.216	27.909	-11.235
90	504	41.100	38.223	42.459	21.879
120	781	41.837	31.237	49.428	25.605

Age	Obs	Model 1	Model 2	Model 3	Model 4
20	742	14.321	21.182	44.394	0.446
40	656	26.695	29.573	44.571	23.369
60	334	39.820	21.933	19.949	18.750
80	232	59.307	29.193	17.070	3.549

Residual Plots for each model by DBH class were examined to detect any model biases (see Figure 2). It would appear that model 3 over predicts the growth of smaller trees and under predicts the growth of larger trees.

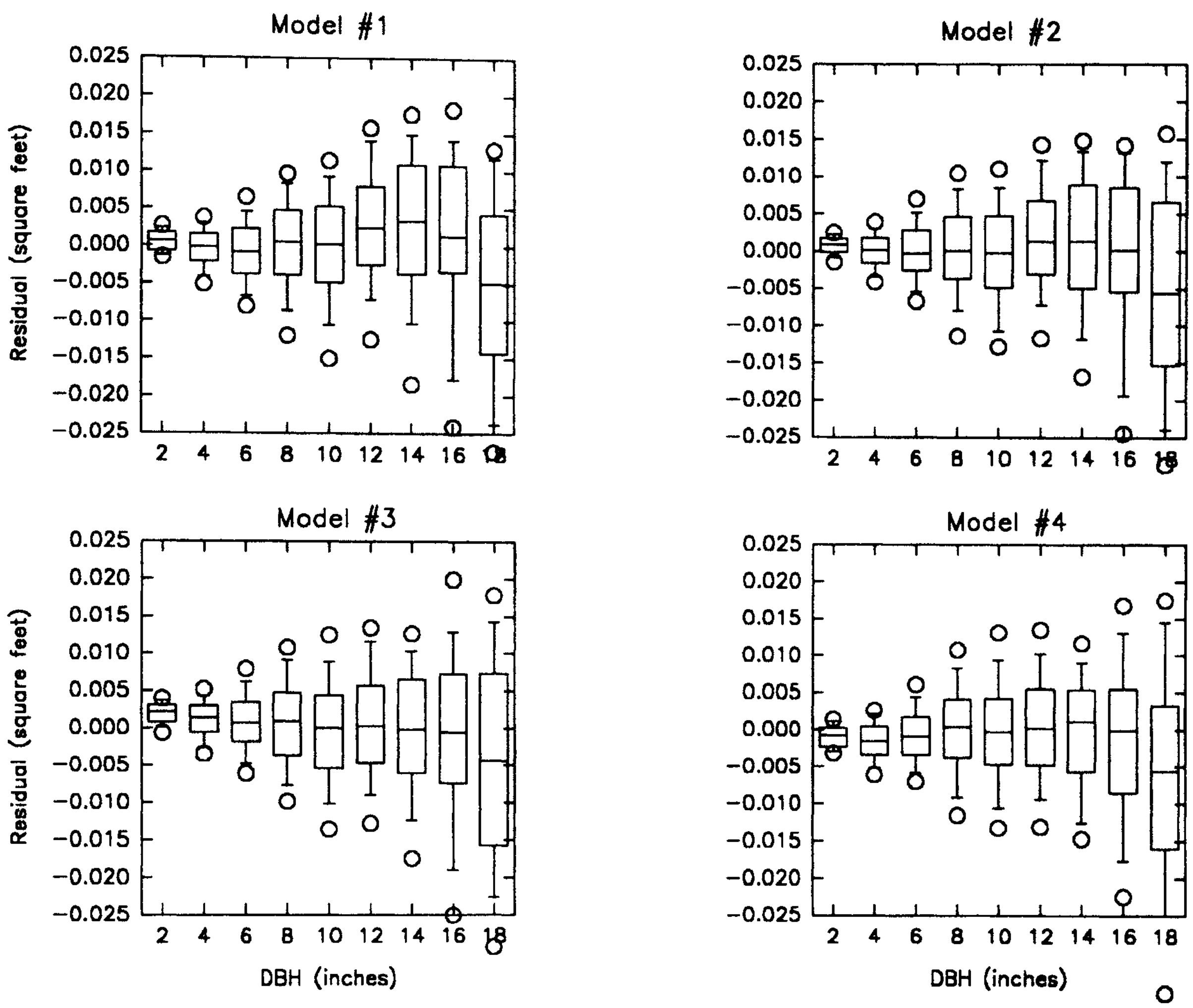

Figure 2. Plots of Residuals vs. DBH Class for all Models When Evaluated with the Independent Data

While all models under estimate the growth of the largest trees (18+ inches DBH), this under prediction is not entirely surprising since these trees were under-represented in the initial data set (see Figure 3). Model 4 appears to under predict the growth of small trees, but performs reasonably well throughout the range of larger diameters. Model 2 showed the least bias based on this criterion.

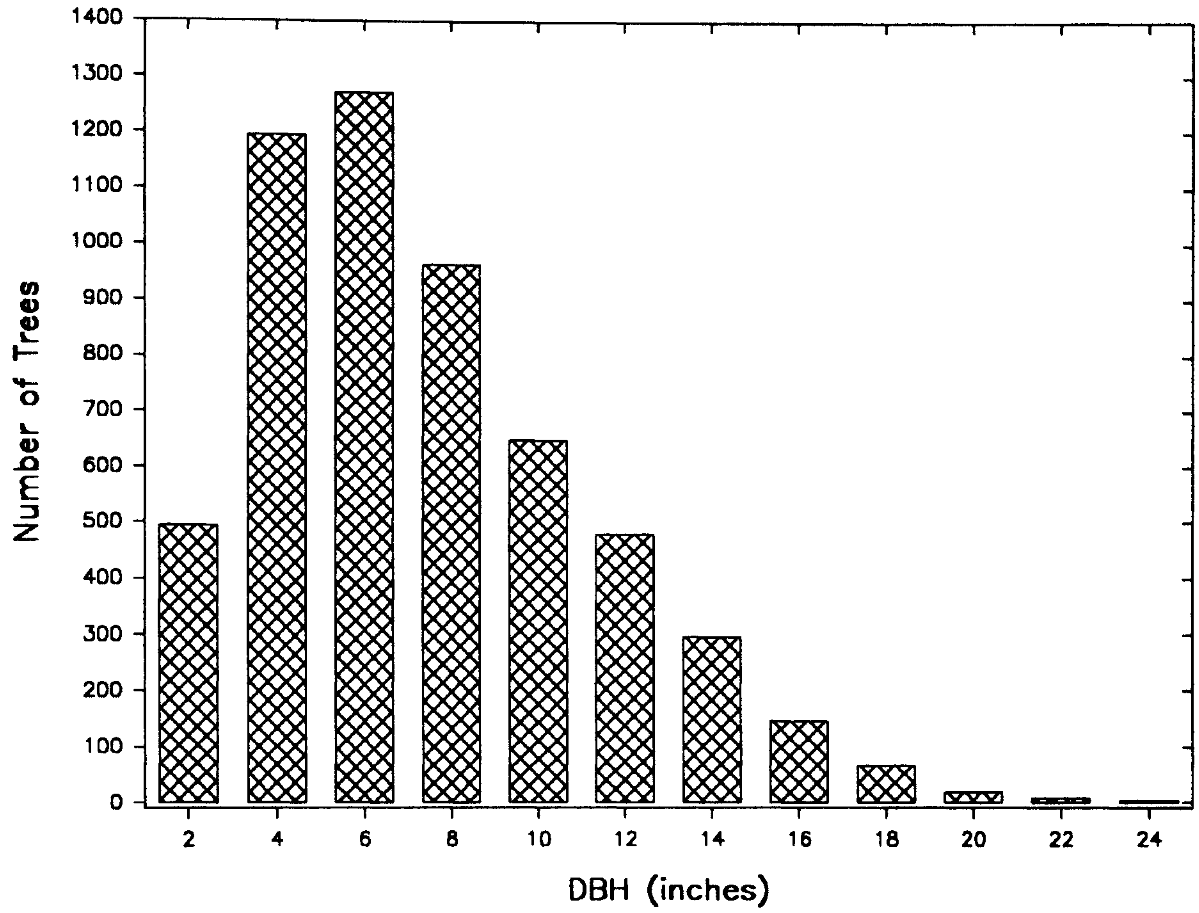

Figure 3. Number of Trees by DBH Class for the Calibration Data Used to Estimate Parameters for All Models.

Residual plots were also examined by site index classes (see figure 4). Model 4 appears to over estimate the growth on higher quality sites and under estimate the growth on low quality sites, while the other models seem to perform comparably based on this criterion.

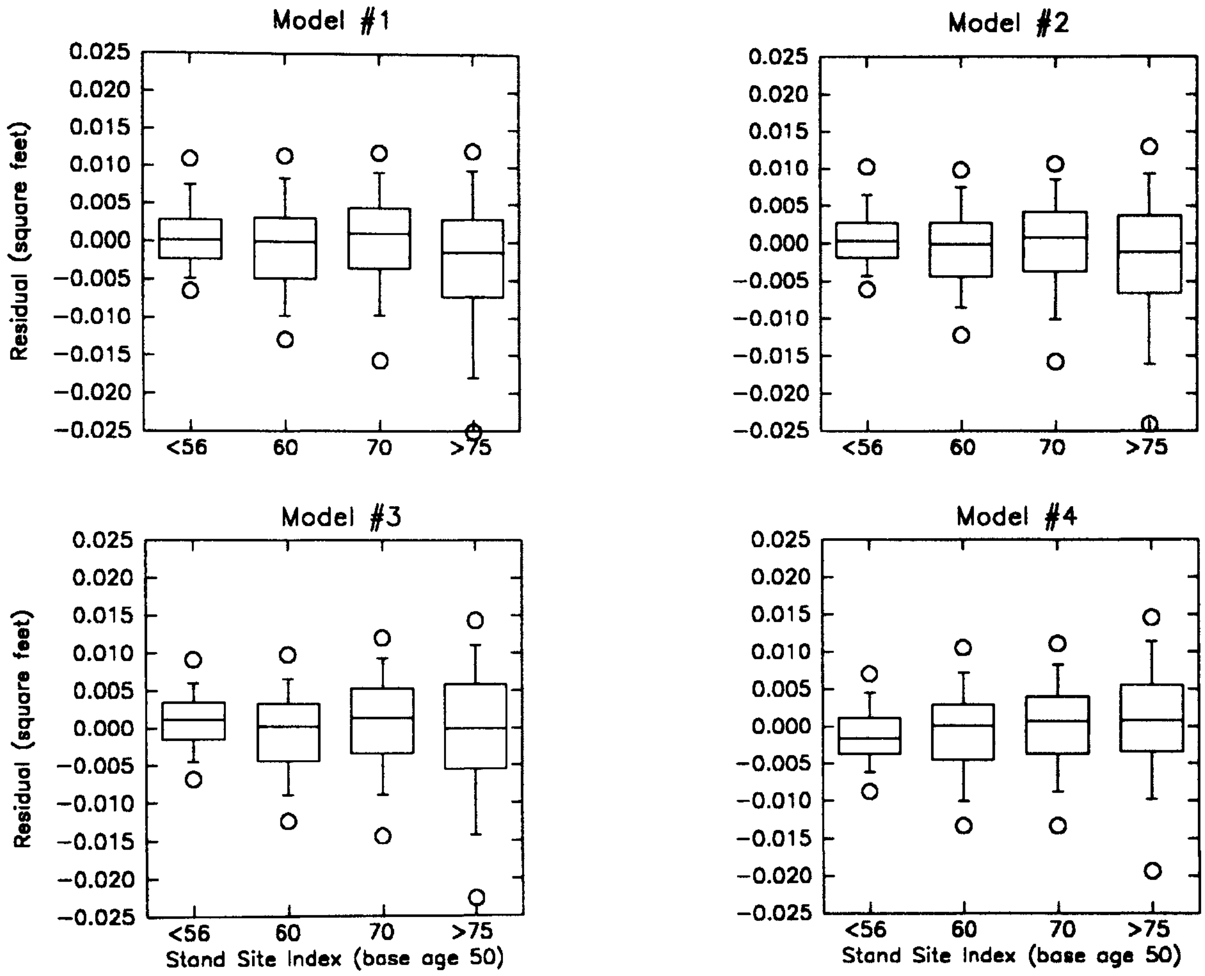

Figure 4. Plot of Residuals vs. Site Index for all Models When Evaluated Using the Independent Data.

Residual plots were examined by stand density levels (see figure 5). Model 3 appears to perform reasonably well throughout the range of density levels while the other models appear to under predict the growth in lower density stands.

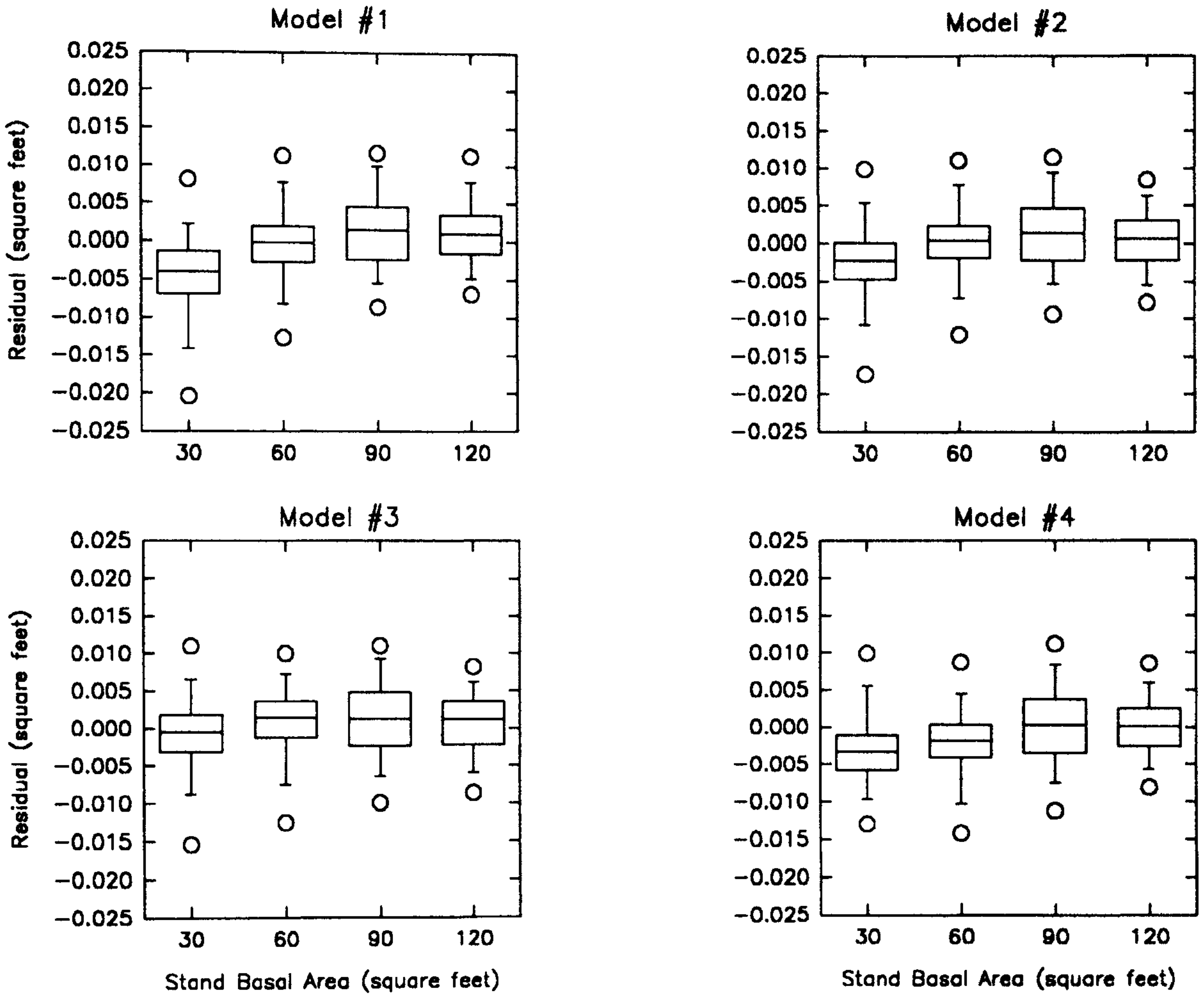

Figure 5. Plots of Residuals vs. Stand Density for all Models When Evaluated Using the Independent Data.

Residual plots were next examined by age classes (see figure 6). Model 1 appears to over predict growth of older trees while, models 3 and 4 tend to under estimate the growth of older trees. Model 2 performs well throughout the range of ages.

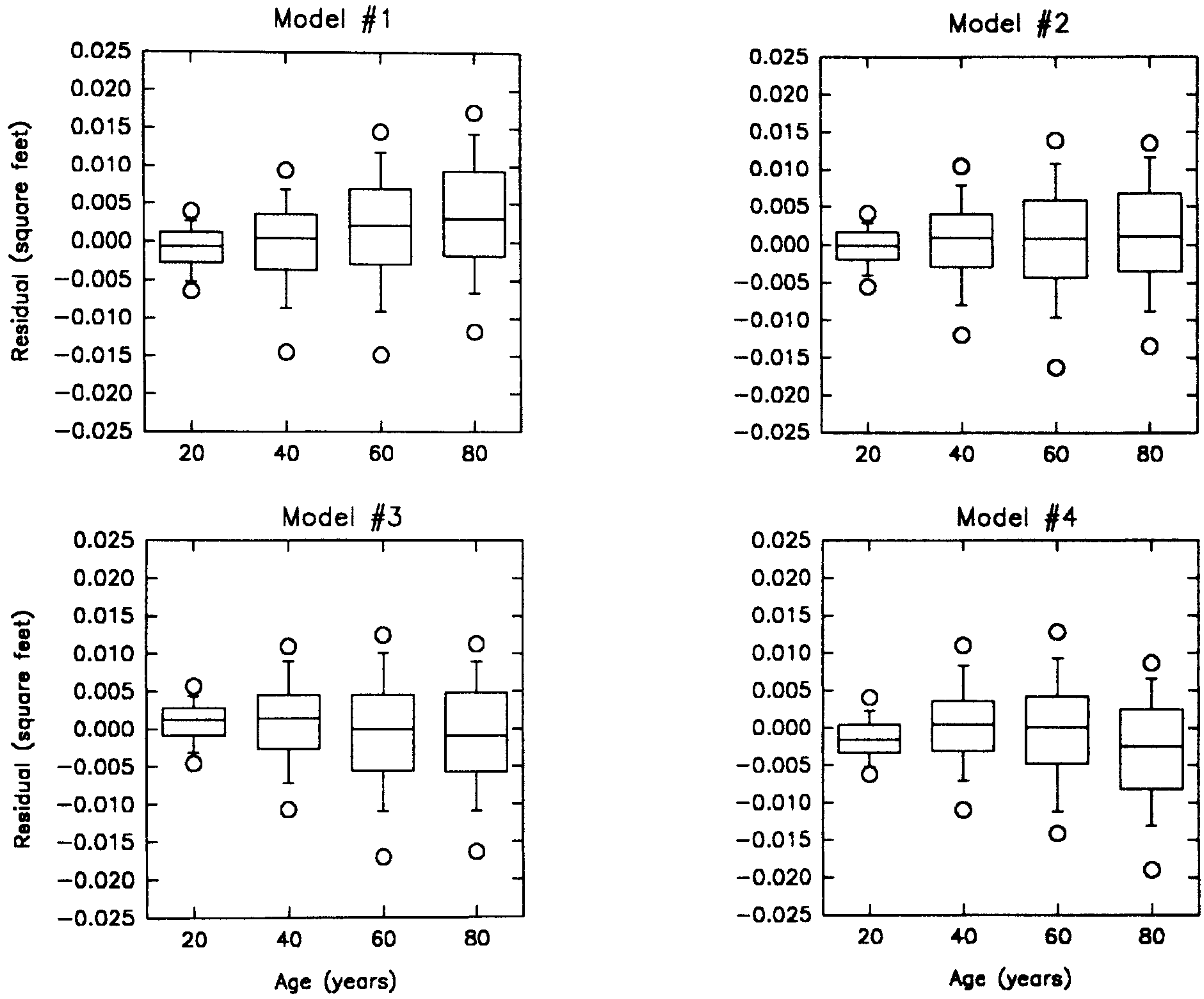

Figure 6. Plots of Residuals vs. Age for all Models When Evaluated Using the Independent Data

Fit Index for each of the models was also computed for the validation data set (see Table XIV). Model 2 had the highest fit index explaining over 62 percent of the variation in growth. Model 1 had the lowest fit index explaining under 57 percent of the variation in growth.

TABLE XIV

FIT STATISTICS FOR MODELS WHEN EVALUATED USING THE INDEPENDENT DATA

Fit Statistic	Model 1	Model 2	Model 3	Model 4
Fit Index	0.56308	0.62625	0.62435	0.61916
Mean Square Error	0.000041	0.000041	0.000041	0.000042
Average Deviation	0.000472	0.000101	0.000414	-0.000836
Average % Deviation	28.105	25.059	37.068	11.582
Mean Absolute Deviation	0.004795	0.004398	0.004533	0.004518

CHAPTER VI

CONCLUSIONS

Based on the results of the validation tests as well as other model characteristics such as structure, logic, and asymptotic tree size, it appears that model 2 is the best overall individual tree basal area growth model. Models 1, 3, and 4 were not selected because all appear to be biased with respect to DBH and age, variables which are easily affected by management practices. Efforts to remove this bias by altering the independent variables and/or adding parameters were not successful. Model #2 appears to under predict growth for stands with less than 45 square feet of basal area per acre. Stands of this density would be rare under normal management regimes, but this condition should be considered when the situation arises. The model performs well against variables which can be controlled by the forest manager such as DBH or age, and therefore should be helpful in decision making.

Model 2 was refitted by combining the original data with the validation set using the iterative procedure discussed earlier and, the following was obtained:

$$AABAG = \frac{\beta_1 \cdot B^{\beta_2} - \beta_1/M^{(1-\beta_2)} \cdot B}{1+\exp(\beta_3 + \beta_4 \cdot BA + \beta_5 \cdot AGE + \beta_6 \cdot BAL).}$$

Where:

AABAG = average annual basal area growth (ft^2),
B = basal area of individual tree (ft^2),
BA = stand basal area (ft^2/acre),
SI = site index (base age 50),
AGE = stand age (years),
BAL = basal area of all trees in the stand as large or larger than the subject tree (ft^2/acre), and
M = 7.068384.

The model explained 60.9 percent of the variation in individual tree basal area growth for the entire data set with a mean square error of 0.000044. Parameter estimates are given in Table XV below.

TABLE XV

PARAMETER ESTIMATES FOR FINAL MODEL WHEN FITTED TO THE ENTIRE DATA SET

Parameter	Estimate	Standard Deviation
ß1	0.104657732	0.00274612
ß2	0.673412127	0.01006582
ß3	-1.95364768	0.05087055
ß4	0.008418773	0.00039880
ß5	0.012151127	0.00054430
ß6	0.013562611	0.00038365

The average deviations were calculated by DBH class for the final model when fitted to the entire dataset and the results are presented in figure 7. The model has relatively high positive deviations for two-inch DBH trees. However, the deviations for all other DBH classes are relatively low.

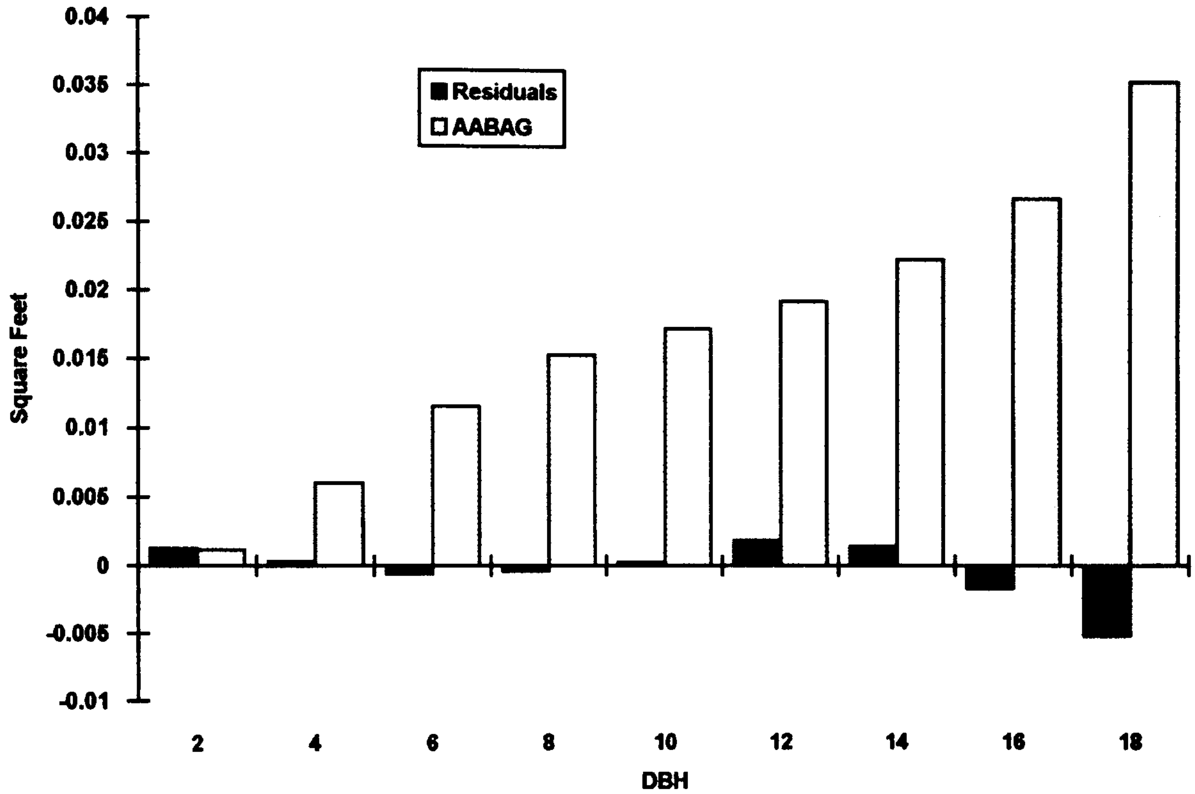

Figure 7. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by DBH Class for the Final Model When fitted to the Entire Data.

Average deviations were calculated by site index class for the final model when fitted to the entire dataset and the results are presented in figure 8. The negative average deviations represent approximately eleven percent of the mean average annual individual tree basal area growth in stands with site indices greater than seventy-five feet.

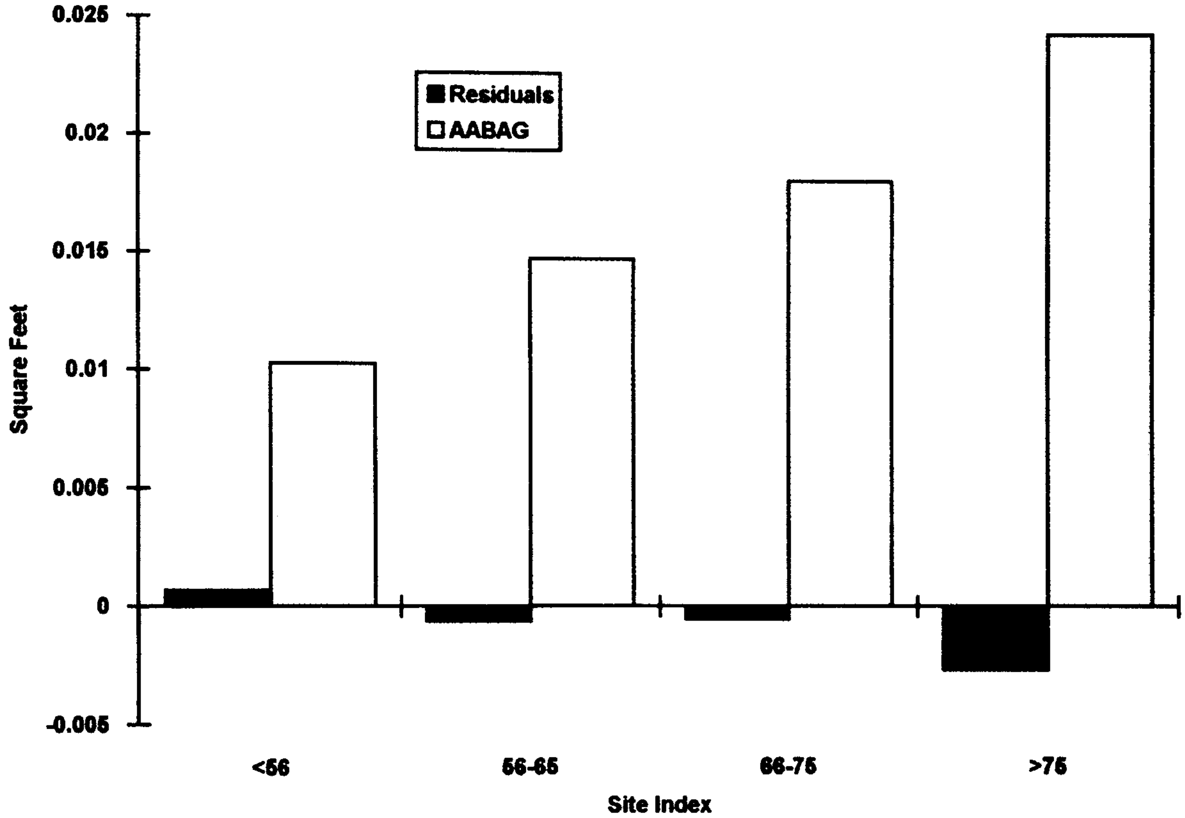

Figure 8. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by Site Index Class for the Final Model When Fitted to the Entire Data.

Average deviations by age class were calculated for the final model when fitted to the entire dataset and the results are presented in figure 9. The average deviation for trees in stands between 50 and 70 years old represent less than six-percent of the mean average annual individual tree basal area growth of trees in those stands. The model performs well throughout the range of the data with relativley low average deviations across all age classes.

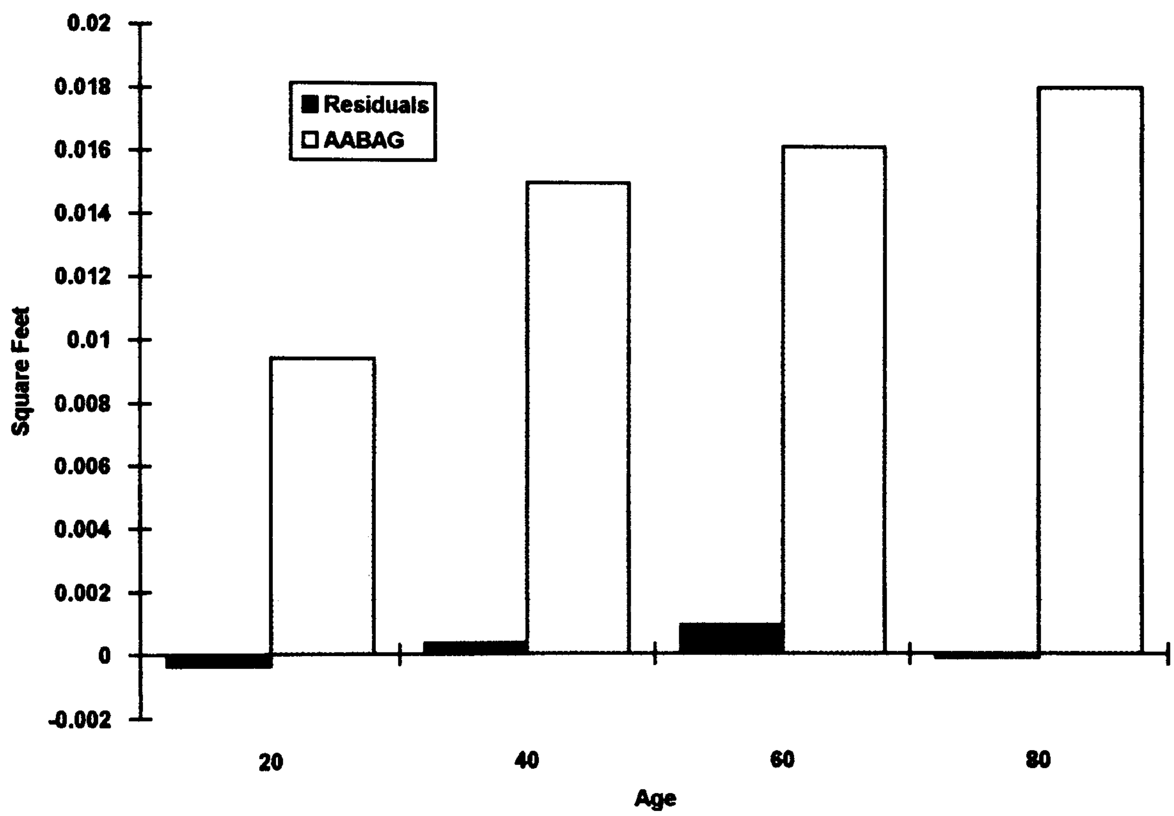

Figure 9. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by Age Class for the Final Model When Fitted to the Entire Data.

Average deviations by stand density class were calculated for the final model when fitted to the entire dataset and the results are presented in figure 10. The negative average deviations for the lowest stand density level indicate that the model tends to under estimate the growth of trees in stands with less than 45 square feet of basal area. Even so the average deviations represent less than six percent of the mean average annual individual tree basal area growth for trees in these low density stands, and less than three percent for trees across the remaining range of densities.

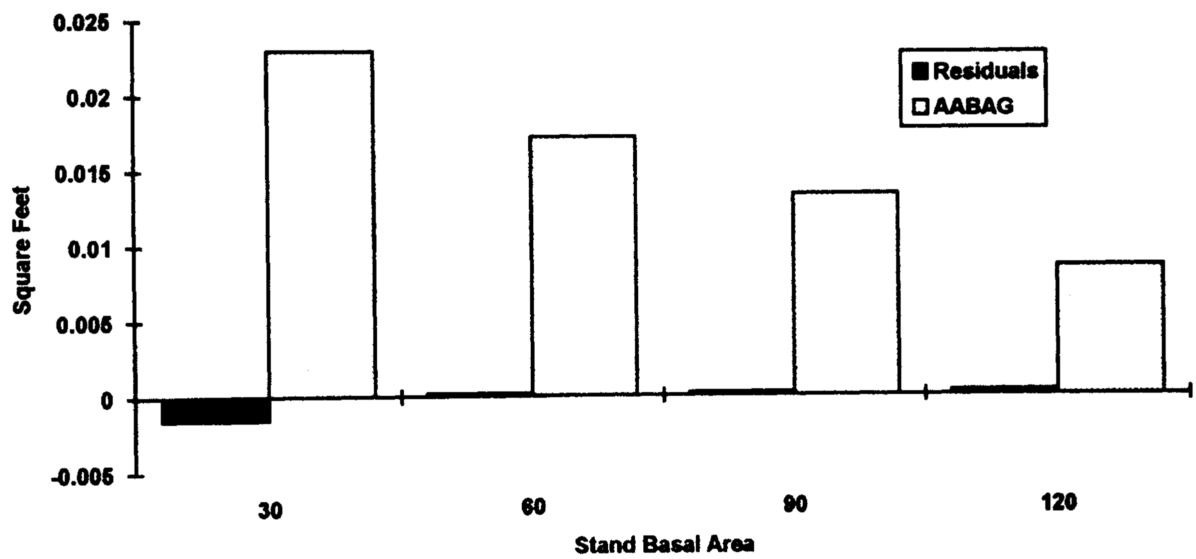

Figure 10. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by Stand Density for the Final Model When Fitted to the Entire Data.

The data used in this study were predominantly from young stands in which basal area growth had not yet culminated, therefore care should be taken when applying the model to mature stands. Also, the model may not perform well at young ages on sites possessing site index greater than 65 feet at 50 years, because there were very few plots in stands of this type. Although care was taken to develop a model which would respond logically to a wide range of conditions, there is no guarantee that the model will produce reliable results when applied outside the range of the study data. Users should refer to Table II, page 27 to verify they are operating within this range.

The equation presented can be used in conjunction with mortality, DBH-height, and individual tree volume models to predict the growth and yield of natural even-aged stands of shortleaf pine throughout Eastern Oklahoma and Western Arkansas. Data are still being collected from the study plots and future measurements may provide better working models. However, the model presented here should be helpful to forest managers throughout eastern Oklahoma and western Arkansas for natural even-aged stands of shortleaf pine, and represents the first step towards the development of a comprehensive growth and yield model for natural even-aged stands of shortleaf pine in eastern Oklahoma and western Arkansas.

LITERATURE CITED

Amateis, R.L., H.E. Burkhart, and T.A. Walsh. 1989. Diameter increment and survival equations for loblolly pine trees growing in thinned and unthinned plantations on cutover, site-prepared lands. Southern Journal of Applied Forestry. 13(4): 170-174.

American Forestry Association. 1992. National register of big trees: The champions of 750-plus species across America. 47 pp.

Belcher, D.W. , M.R. Holdaway, and G.J. Brand. 1982. A description of STEMS. North Central Experiment Station. USDA For. Ser. Gen. Tech. Rep. NC-79. 18 p.

Biging, G.S. and M. Dobbertin. 1992. A comparison of distance-dependent competition measures for height and basal area growth of individual conifer trees. Forest Science. 38(3): 695-720.

Birdsey, R.A. and D.M. May. 1988. Timber resources of east Oklahoma. USDA For. Serv. Res. Bull. SO-135.

Bolton, R.K., and R.S. Meldahl. 1990. Design and development of a multipurpose forest projection system for southern forests. Auburn University, Alabama: Auburn University. Bull. 603. 51 pp.

Brinkman, K.A., N.F. Rogers, and S.F. Gingrich. 1965. Shortleaf pine in Missouri: stand density affects yield. Columbus, Ohio: Central States Forest Experiment Station. USDA For. Serv. Res. Pap. CS-14.

Burkhart, H.E., K.D. Farrar, R.L. Amateis, and R.F. Daniels. 1987. Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas . Blacksburg, Virginia: School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University. FWS-1-87. 47 pp.

Clutter, J.L., J.C. Fortson, L.V. Pienaar, G.H. Brister, and R.L. Bailey. 1983. Timber management; A quantitative approach. New York, NY: John Wiley and Sons. 333 pp.

Daniels, R. F., and H.E. Burkhart. 1975. Simulation of individual tree growth and stand development in managed loblolly pine plantations. . Blacksburg, Virginia: School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University. FWS-5-75. 69 pp.

Daniels, R.F., H.E. Burkhart, G.D. Spittle, and G.L. Somers. 1979. Methods for modeling individual tree growth and stand development in seeded loblolly pine plantations. Blacksburg, Virginia: School of Forestry and Wildlife Resources Virginia Polytechnic Institute and State University. Pub. FWS-1-79. 50 pp.

Davis, L.S., and K.N. Johnson. 1987. Forest management. New York, NY: McGraw-Hill inc.. 790 pp.

Dudek, A., and A.R. Ek. 1980. A bibliography of worldwide literature on individual tree based forest stand growth models. St. Paul, Minnesota: College of Forestry, University of Minnesota. Staff Paper No. 12. 33 pp.

Graney, D.L., and H.E. Burkhart. 1973. Polymorphic site index curves for shortleaf pine in the Ouachita mountains. : USDA For. Serv. Res. Pap. SO-85. 12 pp.

Hahn, J.T., and R.A. Leary. 1979. Potential diameter growth functions. Pages 22-26 in: A generalized forest growth projection system applied to the Lake States Region. USDA For. Serv. Gen. Tech. Rep. NC-49.

Harlow, W.M., E.S. Harrar, and F.M. White. 1978. Textbook of dendrology. New York, NY: McGraw-Hill Inc. 510 pp.

Hilt, D.E., and M.E. Dale,. 1982. Height prediction equations for even-aged upland oak stands. Northeastern Forest Experiment station: USDA For. Serv. Res. Pap. NE-493. 9 pp.

Krajicek, J.E., K.A. Brinkman, and S.F. Gingrich. 1961. Crown competition - a measure of stand density. Forest Science 7(1): 35-42.

Leary, R.A., and M.R. Holdaway. 1979. Modifier function. Pages 31-38 in:A generalized forest growth projection system applied to the Lake State region. USDA For. Serv. Gen. Tech. Rep. NC-49.

Leary, R.A., M.R. Holdaway, and J.T. Hahn. 1979. Diameter growth allocation rule. Pages 39-46 in:A generalized forest growth projection system applied to the Lake State region. USDA For. Serv. Gen. Tech. Rep. NC-49.

Lynch T.B., P.A. Murphy, and E.R. Lawson. 1991. Stand volume equations for managed natural even-aged shortleaf pine in eastern Oklahoma and western Arkansas. Stillwater, Oklahoma: Agricultural Experiment Station Division of Agriculture Oklahoma State University. P-921. 12 pp.

Maple, W.R., and C. Mesavage. 1958. Remarkable shortleaf pine stand. Journal of Forestry 56(4): 290-291.

Martin, G.L., and A.R. Ek. 1984. A comparison of competition measures and growth models for predicting plantation red pine diameter and height growth. Forest Science 30(3): 731-743.

McWilliams, W.H., R.M. Sheffield, M.H. Hansen, and T.W. Birch. 1986. The shortleaf resource. Ed. P.A. Murphy. Pages 1-8 In: Proc. Symposium on the Shortleaf Pine Ecosystem. Little Rock, Ark. Monticello, Ark.: Arkansas Cooperative Extension Service.

Moser, J.W. Jr., O.F. Hall. 1969. Deriving growth and yield functions For uneven-aged forest stands. Forest Science 15(2): 183-188.

Murphy, P.A. 1982. Sawtimber growth and yield for natural, even-aged stands of shortleaf pine in the west gulf region. USDA For. Serv. Res. Pap. SO-181. 13 pp.

Murphy, P.A. 1986. Growth and yield of shortleaf pine. Ed. P.A. Murphy. Pages 159-177 in: Proc. Symposium on the Shortleaf Pine Ecosystem. Little Rock, Arkansas. Monticello, Arkansas: Southern Forest Experiment Station.

Murphy, P.A. 1988. Study plan and progress report: Growth relationships for even-aged stands of shortleaf pine. USDA For. Serv. Southern Forest Experiment Station. 4110 FS-SO- 4106-58 (Problem 3).

Murphy, P.A., and M.G. Shelton. 1993. Progress report: Effects of density, site quality, and maximum diameter on growth and yield of uneven-aged loblolly pine stands. Monticello, AR:FS-SO-4106-36 (Problem 4).

Murphy, P.A., and R.C. Beltz. 1981. Growth and yield of shortleaf pine in the West Gulf Region. Southern Forest Experiment Station. USDA For. Serv. Res. Pap. SO-169. 15 pp.

Murphy, P.A., E.R. Lawson, and T.B. Lynch. 1992. Basal area and volume development of natural even-aged shortleaf pine stands in the Ouachita mountains. Southern Journal of Applied Forestry 16(1): 30-34.

Quick, H., J. Kush, and R.S. Meldahl. 199_. Basal area growth of individual trees model derived from a regional longleaf pine study. Forest Science (in review).

Rogers, R., and I.L. Sander. 1984. Intermediate thinnings in a Missouri shortleaf pine stand: a 30-year history. Ed. E. Shoulders. Pages 216-219 In: Proc. Third Biennial Southern Silvicultural Research Conference. Atlanta, Georgia. New Orleans, Louisiana: Southern Forest Experiment Station. USDA For. Serv. Gen. Tech. Rep. SO-54.

Sander, I.L., and R. Rogers. 1979. Growth and yield of shortleaf pine in Missouri: 21-year results from a thinning study. Pages 14-27 in: Symposium for the Management of Pines of the Interior South. Knoxville, Tennessee. : USDA For. Serv. Southeastern Area State and Private Forestry. Tech. Pub. SA-TP2.

SAS Institute Inc. 1989. SAS/STAT user's guide, version 6, fourth edition, Volume 2. Cary, North Carolina: SAS Institute Inc.. pp.1135-1194.

Schumacher, F.X., and T.S. Coile. 1960. Growth and yields of natural stands of the southern pines. Durham, NC: School of Forestry and Environmental Studies, Duke University. 115 pp.

Shifley, S.R.. 1987. A generalized system of models forecasting central states tree growth. St. Paul, Minnesota. North Central Experiment Station. USDA For. Serv. Res. Pap. NC-279. 10 pp.

Shifley, S.R., and G.J. Brand. 1984. Chapman-Richards growth function constrained for maximum tree size. Forest Science 30(4): 1066-1070.

Smalley, G.W., and R.L. Bailey. 1974. Yield tables and stand structure for shortleaf pine plantations in Tennessee, Alabama, and Georgia highlands. Southern Forest Experiment Station. USDA For. Serv. Res. Pap. SO-97. 58 pp.

Smith, K.L. 1986. Historical perspective. Ed. P.A. Murphy. Pages 1-8 In: Proc. Symposium on the Shortleaf Pine Ecosystem. Little Rock, Ark. Monticello, Ark.: Arkansas Cooperative Extension Service.

Smith, W.R., R.M. Farrar, P.A. Murphy, J. Yeiser, R. Meldahl, and J. Kusk. 1992. Crown and basal area relationships of open-grown southern pines for modeling competition and growth. Canadian Journal of Forest Research 22(3): 341-347.

Trimble, J.L., and C.R. Shriner. 1981. Inventory of United States growth models. Oak Ridge, Tennessee. Environmental Science Division, Oak Ridge National Laboratory. 133 pp.

USDA Forest Service. 1929. Miscellaneous publication no. 50. : USDA For. Serv.. 202 pp.

van Hees, W.S. 1980. Arkansas forests: trends and perspectives. Monticello, Arkansas: Southern Forest Experiment Station. USDA For. Serv. Res. Bull. SO-77. 32 pp.

Wensel, L.C., P.J. Daugherty, and W.J. Meerschaert. 1986. CACTOS user's guide: the California conifer timber output simulator. Oakland, California. Division of Agriculture and Natural Resources, University of California. Bull. 1920.

Wensel, L.C., W.J. Meerschaert, and G.S. Biging. 1987. Tree height and diameter growth models for northern California conifers. Hilgardia 55(8): 1-19.

West, P.W. 1979. Use of diameter increment and basal area increment in tree growth studies. Canadian Journal of Forest Research 10(1): 71-77.

Willet, R.L. 1986. Foreword. Ed. P.A. Murphy. Pages iii-iv In: Proc. Symposium on the Shortleaf Pine Ecosystem. Little Rock, Arkansas. Monticello, Arkansas. USDA For. Serv. Southern Forest Experiment Station.

Williston, H.L. 1975. Selected bibliography on growth and yield of the four major southern pines. USDA For. Serv., State and Private Forestry, Southeast Area. 27 pp.

Wykoff, W.R.. 1986. Supplement to the user's guide for the stand prognosis model: Version 5.0. USDA For. Serv. Gen. Tech. Rep. INT-218. 36 pp.

Wykoff, W.R.. 1990. A basal area increment model for individual conifers in the northern rocky mountains. Forest Science 36(4): 1077-1104.

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Master of Science

Thesis: A DISTANCE-INDEPENDENT INDIVIDUAL TREE BASAL AREA GROWTH MODEL FOR NATURAL EVEN-AGED STANDS OF SHORTLEAF PINE IN EASTERN OKLAHOMA AND WESTERN ARKANSAS

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